A STATISTICAL ANALYSIS OF THE DEMAND FOR LIQUOR IN SWEDEN

A STUDY OF THE DEMAND FOR A RATIONED COMMODITY

BY

STEN MALMQUIST

UPPSALA 1948
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Inaugural Dissertation

by

STEN MALMQUIST
FIL. LIC., SMÅL.

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Preface

Sweden's unique system of liquor rationing provides a wealth of statistical material illustrating different movements, trends etc. in liquor consumption. Using this statistical material as a basis, this paper sets out to analyse the development of consumption from the point of view of demand analysis. Principal stress has been laid on the fact that the commodity under consideration is a rationed commodity.

I should like to express my most sincere thanks to Professor *Herman Wold* for his unfailing and inspiring interest in my work, and the inestimable value of our discussions together.

I am also indebted to my friends and colleagues at the Institute of Statistics for having read the MS. and pointed out a number of omissions.

I extend my thanks to Mr. *Edric Hille* for undertaking the work of translating the MS.

For financial assistance, I am indebted to the »Statens Samhällsvetenskapliga forskningsråd».

Uppsala, October 1948.

Sten Malmquist.

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CHAPTER I

In Sweden, every person wishing to buy wines or spirits other than for immediate consumption on a licensed premises (such as hotel, restaurant etc.) must first apply for a ration book, called in Swedish »motbok». Application for a ration book is made to the »system company» in the area in which the person is residing. A »system company» denotes a company with a government monopoly to sell wines and spirits within a specified area of the country. On receiving an application for a ration book, the »system company» first decides whether or not the person in question is entitled to be supplied with liquor, and if so, the quantity of liquor he or she may buy each month. The size of the ration varies considerably, being based on the applicant's age, marital status, income and other special factors. For spirits, the maximum ration is four litres per month, while for wine there is in principle no maximum ration.[1]

§ 1 of this chapter will deal briefly with the principal changes in liquor restrictions as they affect sales to persons holding ration books, in other words, bottle store sales.

Information relating to the sale of intoxicating beverages from 1914 onwards is contained in an annual publication included in Sweden's official statistics.[2] Paragraphs 2 and 3 give a summary of ration book statistics, together with information concerning the sale of intoxicating beverages. Paragraphs

[1] An account of the control system is given by M. Marcus in, »The Liquor Control System in Sweden». Stockholm 1946.

[2] 1914—18: »Accispliktiga näringar. Brännvinsförsäljningen.» Sveriges officiella statistik.

1919—22: »Accispliktiga näringar. Rusdrycksförsäljningen.» S. o. s.

1923— : »Rusdrycksförsäljningen m. m.» S. o. s.

4 and 5 analyse, by means of ordinary regression technique, the changes in the quantities of wines and spirits sold to ration book holders during the period 1923-40.

An account of the theory of demand and regression analysis is given in a paper by Wold.[1] The regression analysis developed by him is also adopted here.

The relationship between the variations in quantity, and certain other factors, is given in the form of an elasticity coefficient. One can say that the elasticity of quantity with regard to price, or price elasticity, is, let us say E_{KP}, if a relatively small change in price, say C %, produces a change in quantity amounting to E_{KP} C %. As an increase in the price of a commodity generally results in fewer purchases being made of that commodity, price elasticity is generally negative. If, for example, a price increase of 1 % results in a decrease of about 0.5 % in the amount of the commodity purchased, the price elasticity will then be — 0.5. If the price elasticity of a commodity is — 1, the demand for the commodity is called normal-elastic. If, on the other hand, the price elasticity is numerically greater or smaller than 1, the demand is said to be over-elastic or under-elastic respectively. As a rule, an under-elastic demand is characteristic of so called necessities, while the demand for luxury goods is generally over-elastic. The under- and over-elastic demand respectively is also characterised by the fact that the expenditure on the commodity in question increases or decreases respectively when the price is raised. With normal-elastic demand, the quantity bought when the price is raised is decreased in such a way that the expenditure, that is to say, the product of the decreased quantity and the increased price, remains unchanged.

An elasticity coefficient is independent of the units used for expressing the price and quantity in question. For this reason, it is a suitable expression for comparing the demand for different goods.

The following have been chosen as independent variables in

[1] H. Wold: »Efterfrågan på jordbruksprodukter och dess känslighet för pris- och inkomstförändringar» (The Demand for Agricultural Products and its Elasticities with Respect to Price and Income). Stockholm 1940. This paper is henceforth referred to as Wold 1940.

the regression analysis: the price of liquor, consumers' incomes, size of rations, and cost of living index.

The fact that liquor is not a uniform commodity, but consists of a number of different kinds of commodities with varying prices has necessitated the use of alternative price series. Besides the average price obtained by dividing the annual expenditure by the corresponding quantity of liquor sold, two other price variables are used, namely an average price from which a certain effect due to changes in consumption has been eliminated, and the price of the quantitatively most important commodity.

Estimate of income according to taxation returns has been used as the income variable.

In the case where the available income series cannot be regarded as a satisfactory expression of the changes in the income of the consumers, it is possible to calculate the relationship between the income elasticities for two commodities on the basis of the conditions given in § 6, although one does not know the real changes in income.

Regression analysis shows the price elasticities for liquor to be between — 0.2 and — 0.4. The elasticity of the sale of wine with regard to the price of liquor works out at about — 0.6 to — 1.0, according to the alternative methods of calculation.

With the method described in § 6, the income elasticity for bottle store sale of wine works out to be in the order of three times greater than the income elasticity for sale of spirits.

These results would appear to show that the demand for liquor is decidedly under-elastic. In judging the elasticities, however, one must pay regard to the fact that one is dealing with a rationed commodity. In the next chapter we shall examine the significance of an elasticity of this kind.

§ 1. Restrictions on the sale of liquor by bottle stores

Individual control of the sale of liquor by bottle stores was first introduced in Gothenburg, where certain restrictions came into force in October 1912. Other liquor retailing companies gradually followed suit, and from January, 1, 1916, individual control was made obligatory. This control included, »*firstly*,

obligation for a person applying to a company and requesting to be accepted as a customer to identify himself; *secondly*, obligation for the customer to identify himself each time he made a purchase; *thirdly*, noting down the amount sold at each individual transaction and, later, the recording and classification of these notes». The companies had free choice as to whether they wanted to set a limit on the amount purchased or not. In the case of the majority of companies, the actual purchases were checked after they had been made, and abuse resulted in a limitation of purchases or curtailment of the ration card. The lowest maximum quantity, where such was stipulated, was 12 litres per 3-month period, additional purchases being permitted for special purposes, however.

With the object of cutting down the liquor consumption during the conditions prevailing at the time, it was stipulated, among other things, that from November 16, 1916, only 2 litres per month might be bought from the bottle stores. In conjunction with the vote on prohibition, all bottle stores suspended sales from April 24 to May 31, 1917. After this, the maximum ration was set at 2 litres per 3-month period, and it was also stipulated that nobody under 25 might buy liquor from bottle stores. Because of the small amount of liquor in supply, this ration was retained until May 20, 1919. After this date, the companies were permitted to sell the maximum quantity stipulated in the regulations, namely 4 litres per month.

A special excise that was introduced on liquor at the same time, however, had the effect of restricting consumption. According to the legislation covering liquor excise that was in force until the end of 1919, excise on the retail sale of spirits amounted to five Swedish kronor on the first, ten kronor on the second, and twenty kronor on each additional litre of spirits sold to the same person during a three month period. However, persons who had acquired the right to purchase liquor before the beginning of the third calender month were not required to pay excise during the current quarter on the first three litres they purchased, providing they had actually obtained their ration card before the beginning of the second month in the quarter; if they had obtained their ration card after this date they were exempt excise on the first two litres purchased. In

restaurants and such like, there was an excise of five kronor per litre.

During the second world war there were also special restrictions on the sale of liquor. On October 1, 1941, there was a general reduction in rations, persons formerly entitled to 4 litres per month now received only 3, those formerly entitled to 3 receiving 2. In addition, the right to so called »post-date purchases» was cancelled. This had been introduced on October 1, 1938, and what it amounted to was that a person could take out his monthly ration after the end of the month, though not later than six months after. On October 1, 1941, the customary »Christmas litre» was also cancelled — previously it had been the custom to allow certain ration book holders an extra litre for Christmas.

§ 2. The number of ration books and the size of rations

A. The first exact figure for the number of ration cards issued relates to the number of ration cards held on November 10, 1916. On this date there were 794.246 ration card holders. The number increased (following a temporary decrease due to the prohibition of the sale of liquor to persons under the age of 25) to 1,055,451, at the middle of 1920 (Aug. 31, 1920). During 1921 and 1922 the number of ration book holders decreased, chiefly on account of a tightening up of the control on ration books that followed a circular sent out by the Board of Control on July 3, 1920. The increase in the number of ration books from 1922 onwards has been interrupted only by the increase in the price of liquor, the economic depression during 1932 and 1933, and the increase in prices at the end of 1939 and the years following. After a certain reduction in the size of rations in 1941, the number of ration book holders increased, however, and on the 31st. of December 1946 amounted to 1,769,830.

The average number of ration books in relation to the average population between the ages of $20 - \omega$ was 0.272 in 1923, and 0.345 in 1944, an increase of 27 %. The increase in the number of ration book holders is thus considerably greater than can be accounted for by the increase in population.

Number of male ration book holders over 25 expressed as a percentage of the population over 25.

Year	%
1920	65.0
1924	60.4
1930	64.5
1939	68.5
1946	73.3

From 1923 to 1932 the number of female ration book holders in relation to the total number of ration book holders increased from 8.1 % to 10.9 % and since 1932 the percentage has remained practically constant.

From 1923 to 1931 the number of ration book holders under 25 was insignificant. After 1933 the number increased, following the new regulations for sale of liquor introduced in October 1938 when the normal minimum age at which a person might obtain a ration book was set at 21.

Number of ration book holders under 25 in ‰ of the total number of ration book holders.

	Men	Women		Men	Women
Oct. 1, 1925	6.5	0.6	Dec. 31, 1941	19.3	0.1
Dec. 31, 1927	5.1	0.3	Dec. 31, 1942	12.9	0.0
Dec. 31, 1933	6.4	0.1	Dec. 31, 1943	7.0	0.0
March 31, 1939	24.5	0.4	Dec. 31, 1944	7.3	0.0
Dec. 31, 1939	24.5	0.4	Dec. 31, 1945	7.3	0.0
Dec. 31, 1940	20.2	0.2	Dec. 31, 1946	8.7	0.1

B. From 1920 there are figures available for the number of ration book holders entitled to »limited purchases», that is to say, less than 4 litres per month. On June 30, 1920, ration books of this type amounted to 24.4 % of the total. Chiefly as a result of the circular sent out by the Control Board, which

has already been alluded to, the following years saw a drastic reduction in the number of ration books entitling the holder to 4 litres per month. Thus, on Dec. 31, 1922, the number of »limited purchase» ration books amounted to 68.9 % of the total number of ration books valid at that date, while among male ration book holders alone 66.3 % had »limited purchase» books. These percentages have since increased, amounting on Dec. 31, 1940, to 81.6 % and 79.4 % respectively. Assuming that all ration book holders under 25 had »limited purchase» books, then of the remaining male ration book holders, 77.2 % had limited purchase books on Dec. 31, 1933 (77.3 % of all male ration book holders) and 79.0 % on Dec. 31, 1940. The increase in the number of ration book holders entitled to »limited purchases» cannot be explained, therefore, by an increase in the relative number of female ration book holders, and ration book holders under 25.

1923 is the first year for which there are official statistics for the average ration per book. The average ration has been continually on the decrease; in 1923 it was 2.50 litres per month, in 1941, 2.10 litres per month. The reduction of certain rations in October 1941 brought the average monthly ration down to 1.76 litres.

§ 3. Sale of wines, spirits and beers

On the basis of figures for import and export, as well as certain information concerning supplies in stock, estimates have been made of the amount of spirits sold in earlier years. The centralisation of sales increased the possibilities in this respect. From 1917 onwards, all legal sale of spirits is shown in the statistics kept by the Board of Control. Figures for earlier years refer to the number of litres of spirits at 40 % strength, distillates such as brandy and 96 % alcohol being reduced to 50 % strength, other kinds of liquor at their actual strengths.

From 1919 onwards, Swedish official statistics record the exact amount of wine sold annually. In estimating the quantities sold in earlier years, use has been made of figures for the amount of wine imported and estimates of the home pro-

Consumption of liquor reduced to 40 % strength, in millions of litres.

Years	Mill. l.	Per inhab.
1861—65	42.6	10.7
66—70	37.0	8.9
71—75	50.6	11.8
76—80	45.5	10.1
81—85	37.9	8.0
86—90	33.8	7.1
91—95	33.0	6.9
91—00	40.4	8.1
1901—05	39.6	7.6
06—10	36.8	6.8
1911	36.9	6.6
12	38.1	6.8
13	38.7	6.9
14	37.2	6.6
15	36.8	6.5
16	29.9	5.2
17	9.5	1.7
18	7.2	1.2
19	13.5	2.3
20	35.2	6.0
21	28.4	4.8

duction. For beer, there are ligures for production years from 1923/24 onwards, giving the taxable quantities sold by the breweries.

The figures in the table referring to quantities sold by restaurants and other licensed premises are for bulk sales to such establishments, not the amounts actually sold to customers. Sale of liquor by licensed premises is also subject to restrictions of various kinds.

From the percentage fluctuations for the quantities sold, given in the table below, it will be seen that the fluctuations are parallel, and are greatest for the sale of wine. It will also be

seen that the fluctuations in the sale of liquor by bottle stores and the fluctuations in the sale of beer are roughly of the same magnitude.

Percentage fluctuations at different periods in the quantities of spirits, wines and beers sold.

Years	1	2	3	4	5	6
1920-22	— 31	29	- 31	- - 40	— 67	—
23-39	+ 25	+ 6	+144	+ 26	+ 91	—
24-30	— 16	— 1	+ 95	+ 13	+ 78	+ 25
30-31	+ 3	— 0	— 0	— 4	— 11	— 3
30-33	— 15	- 19	— 38	— 33	— 31	— 25
33-38	+ 27	+ 12	+ 70	+46*	+112*	+ 30
38-40	20	— 25	— 65	—16**	—34**	— 10
41-46	+ 23	+ 8	+170	+ 56	+ 5	+ 20

* 1933/(April 1938—March 1939).
** (April 1938—March 1939)/40.

Column 1. Total sale of liquor (by bottle stores).
» 2. Sale of liquor per ration book holder.
» 3. Total sale of wine (by bottle stores).
» 4. Total sale of liquor (by licensed premises).
» 5. Total sale of wine (by licensed premises).
» 6. Total sale of class II beer.

Sale of spirits, wines and beers, 1919-1946.

	1**	2	3	4	5	6	7	8	9	10	11
1919	931	263		12.16	13.06	3.39	2.28	2050			2.71
20	1043	290		33.98	32.58	2.67	4.20	1341			5.64
21	1046	287		28.79	27.52	2.15	3.07	591			4.76
22	1014	275		23.42	23.10	1.89	2.51	449			3.88
23	1014	272	2.50	23.70	23.37	2.21	2.81	553			4.05
24	1026	273	2.50	25.56	24.91	2.73	3.12	594	135	31.1	4.31
25	1060	279	2.46	26.72	25.21	3.12	3.17	659	143	32.6	4.61
26	1088	283	2.41	27.11	24.92	3.44	3.10	735	149	33.6	4.68
27	1113	286	2.35	27.55	24.75	3.84	3.11	816	151	33.8	4.79
28	1136	289	2.30	28.08	24.72	4.09	3.29	919	150	33.3	4.83
29	1166	293	2.26	28.79	24.69	4.47	3.43	1034	156	34.3	4.99
30	1202	298	2.23	29.63	24.65	4.54	3.53	1058	169	36.7	5.18
31	1239	303	2.22	30.47	24.59	4.39	3.40	946	164	35.3	5.13
32	1262	305	2.19	28.92	22.92	3.21	2.86	729	153	32.6	4.74
33	1259	300	2.16	25.26	20.06	2.67	2.37	726	126	26.7	4.05
34	1268	298	2.17	26.32	20.76	2.91	2.58	855	128	26.7	4.10
35	1296	301	2.16	27.51	21.23	3.07	2.89	998	132	27.4	4.36
36	1328	306	2.15	28.67	21.59	3.18	3.02	1145	140	28.6	4.38
37	1372	313	2.13	30.14	21.97	3.61	3.27	1312	151	30.6	4.60
38	1427	323	2.13	32.03	22.45	3.99	3.12	1445	164	32.9	4.82
39	1512	339	2.12	31.61	20.91	3.21	3.44	1435	172	34.2	4.83
40	1519	336	2.11	25.67	16.90	1.30	2.96	1021	147	29.1	3.96
41	1520	332	2.10*	25.53	16.80	1.63	3.19	1098	122	24.0	3.77
42	1550	335	1.75	25.82	16.66	2.02	4.11	1086	125	24.4	3.63
43	1588	340	1.73	27.47	17.30	2.40	3.98	986	132	25.5	3.96
44	1629	345	1.70	28.60	17.56	2.86	4.06	920	127	24.4	4.03
45	1677		1.68	29.82	17.78	3.17	4.34	934	138		
46	1734		1.63	31.36	18.09	3.86	4.97	1149	149		

* Reduced to 1.76 for October 1941.

** In thousands of persons.

Column 1. *The average number of ration book holders.* Uncompensated arithmetical average of the number of ration book holders during the 12 months of the year.

Column 2. *The number of ration book holders per 1000 persons between the ages 20—ω.*

Column 3. *The average ration per ration book holder in litres per month.*

Column 4. *Sale of liquor* (through bottle stores) *in millions of litres.* 96 % spirits calculated at 40 % strength. After 1923, the reduction factor 200/86 = 2.3256 has been used, as 86 c.c. of 96 % spirits is reckoned as 2 litres of liquor from a rationing point of view.

Column 5. *The amount of liquor purchased per ration book holder* (litres/year). The figures in column 4 (No. of litres) divided by the average number of ration book holders as shown by column 1.

Column 6. *Sale of wine in litres per ration book holder.*

Column 7. *Sale of liquor to licensed premises in millions of litres.*

Column 8. *Sale of wine to licensed premises in thousands of litres.*

Column 9. *Sale of beer in millions of litres.*

Column 10. *Sale of beer per inhabitant between the ages 15—ω, expressed in litres.*

Column 11. *Total consumption (on basis of alcohol content) per inhabitant between the ages 15—ω, expressed in litres.*

§ 4. Regression analysis of the quantities of liquor sold by bottle stores

The restrictive measures described above which affected the sale of liquor by bottle stores during the period 1917-1920 were of decisive importance for the quantity of liquor sold.

The size of the rations has been presumed to be one of the factors that influence the sale of liquor by bottle stores. During the years 1920-22, after the wartime restrictions had been lifted, there was a drastic reduction — as already mentioned — in the size of the rations. This reduction was of a different character than the reduction of the average ration over the years 1923-40. In all probability, therefore, the changes in the average ration during these two periods has had a different effect on the quantities of liquor sold by the bottle stores (cf. Chap. II § 8). The same thing can be said of the reduction of certain rations in October 1941. As a consequence, we shall study the fluctuations in the sale of liquor by bottle stores from 1923 to 1940, in other words, up to the year preceding the latest reduction in rations.

The factors exercising the greatest influence on the amounts purchased have been assumed to be the price of liquor, the consumers' incomes, the size of the rations, and the prices of other goods (in the widest sense).

A. *Prices*: As a measure of the year to year variations in the nominal liquor prices one can, to begin with, use the fluctuations in the average price obtained by dividing the annual value of sales by the corresponding number of litres sold by the bottle stores. This average price is thus a weighted average value of the prices of the various kinds of liquor, the weights being the quantities sold during the year in question. However, as the term liqour includes a large number of liquors of varying quality and price, it is clear that a price series of this kind is not only an expression of price changes, but also of the variations in the distribution of purchases among the cheaper and more expensive qualities. Official statistics for 1923-38 split up the figures for value and quantity into different main types of liquor. From these figures it is possible to obtain a series for the average price, based on the assumption that the distribution of purchases among the main types of liquor remained unchanged. This price series has been calculated here as a weighted mean of the average prices for certain main kinds of liquor (40 % distilled spirits[1], distilled spirits[2], spirits of other kinds[3], 96 % spirits, French brandy or cognac, whisky, rum and arrack, punch including liqueur and burnt and spiced brandy) for the year in question. The weights consist of the quantities purchased in 1930.

A comparison between these average price series shows that a certain trend towards buying more expensive liquor took place during the period 1923-30. The increase in prices in 1932 and 1933 combined with falling incomes had the reverse effect. It must be pointed out that this comparison gives an idea only of the change in the average price as a result of a redistribution of purchases among different main kinds of liquor, and not as a result of changes in the distribution within these main types.

Yet another price series may be cited, namely the average price of a single type of liquor, 40 % distilled spirits. From

[1] In swedish: »Renat brännvin à 40 %».
[2] In swedish: Renat brännvin».
[3] In swedish: »Brännvin av annat slag».

1923-40 this relatively cheap variety of liquor accounted for about 50 % of all liquor sold by bottle stores.

Finally, it should be pointed out that each marked change in the price of liquor is the result of an increase in tax, the prices of different kinds of liquor being varied by the same amount.

Year	1. P'_N	2.	3. P''_N	4. P'''_N	5.	6. I_R	7. L
1923	4.71	5.87		3.54	4314	644.2	177
24	3.99	5.22		2.88	4474	673.4	174
25	3.84	4.79	3.94	2.80	4569	673.4	176
26	3.74	4.64	3.79	2.78	4683	699.9	172
27	3.72	4.47	3.73	2.75	4779	711.4	171
28	3.68	4.43	3.69	2.70	4865	713.5	172
29	3.59	4.31	3.58	2.65	5150	761.8	170
30	3.56	4.30	3.56	2.65	5272	796.8	165
31	3.48	4.28	3.49	2.65	5019	775.0	159
32	4.13	4.46	4.26	3.45	4648	723.7	157
33	4.92	4.42	5.10	4.30	4502	706.7	154
34	4.97	4.31	5.17	4 35	4863	750.2	154
35	4.99	4.42	5.19	4.35	5355	805.7	156
36	5.00	4.50	5.19	4.35	5680	833.4	158
37	5.01	4.41	5.20	4.35	6237	884.0	162
38	5.10	4.40	5.29	4.47	6690	917.9	166
39	5.76	4.79		5.17	7046	931.0	171
40	8.86	7.30		8.26	7353	859.2	192
41	9.37	8.12		8 64			
42	10.38			9.49			

Column 1. Average price per litre for liquor sold through bottle stores.
Column 2. » » » » » wine » » » »
Column 3. » » » » » liquor » » »
calculated from the distribution of purchases among the main types of liquor in 1930.
Column 4. Average price per litre for distilled 40 % spirits.
Column 5. Estimated income in millions of kronor based on the communal taxations returns.
Column 6. Estimate of real income per inhabitant between the ages of 15 and 65.
Column 7. Cost of living index (1914 = 100).

Incomes: There are no figures available for the incomes of ration book holders. As a basis for estimating the changes in these incomes, it is possible to use income assessments as given by the communal taxation returns (before the 1929 Finance Act) for all persons liable to tax other than Swedish joint-stock companies and joint bank companies.[1] The real value of the incomes has been obtained by dividing by the cost of living index issued by the Social Board.

B. Thee relationship between one variable and certain other variables will be given in the form of a coefficient of elasticity.

The mathematical definition of a price elasticity, for instance, is as follows: Let the quantity, K, be determined by the derivable function

$$K = f(P, Q),$$

where P is the price of the commodity in question and Q denotes all other factors affecting the size of the quantity purchased. The elasticity of the quantity in respect to the price may be written $E_{KP \cdot Q}$ and is defined by

$$E_{KP \cdot Q} = \frac{\delta K}{\delta P} \frac{P}{K} = \frac{\delta \log K}{\delta \log P}$$

Further, let the expenditure for the commodity be represented by O. One then gets

$$\frac{\delta O}{\delta P} = \frac{\delta KP}{\delta P} = K(1 + E).$$

From this relationship one gets the variation in the expenditure for an under-, normal- and over-elastic demand.

In the statistical calculations of the elasticities, one devises a function which describes how the average value of the logarithmical quantity, say $M(\log K)$, varies with the supposed explanatory factors.[2] Let this function be

$$M(\log K) = f^*(\log P, Q).$$

[1] See »Skattetaxeringarna» (Taxation returns), Sveriges officiella statistik.
[2] Wold 1940, § 10.

The price elasticity is defined by

$$E_{KP \cdot Q} = \frac{\delta M(\log K)}{\delta \log P}.$$

For mathematically defined elasticities one has the equation

$$E_{KP \cdot Q} \cdot E_{PK \cdot Q} = 1.$$

The corresponding relations is not always valid for the statistically calculated elasticities.

Suppose the elasticities are constant. Then the relationship is valid only in such cases where the partial coefficient of correlation between the logarithm for the quantity variable and the logarithm for the price variable are equal to 1. If the coefficient of correlation referred to is denoted by $r_{KP \cdot Q}$, one gets

$$E_{KP \cdot \bar{Q}} \cdot E_{PK \cdot \bar{Q}} = r_{KP \cdot \bar{Q}}.$$

In the following regression analysis the logarithms of the variables are thus used.

The logarithm of the average purchase per ration book holder is here taken as the dependent variable, and the logarithms of price, average ration income, and cost of living index respectively are used as the independent variables.

Assuming that the elasticities are constant, we shall endeavour to find the values a_0, a_1, a_2, a_3 and a_4, which minimize the expression

$$S\left(a_0, a_1, a_2, a_3, a_4,\right) =$$

$$= \sum_{\nu=1}^{n} \left(\log K_\nu - a_0 - a_1 \log P_\nu - a_2 \log T_\nu - a_3 \log I_\nu - a_4 \log L_\nu \right)^2,$$

where n = the number of years covered by the observations,
$\log K_\nu$ = the average quantity purchased during the year ν,
$\log P_\nu$ = the logarithm of the price for the year ν,
$\log T_\nu$ = the logarithm of the average ration during the year ν,
$\log I_\nu$ = the logarithm of the income for the year ν,
$\log L_\nu$ = the logarithm of the cost of living index for the year ν,

The coefficients a_1, a_2, a_3 and a_4 are the elasticities of quantity with regard to price, average ration, income and cost of living index, respectively. Henceforth, in cases where other independent variables are also given, these elasticities will be denoted as follows:

$$a_1 = E_{KP \cdot TIL};\ a_2 = E_{KT \cdot PIL};\ a_3 = E_{KI \cdot PTL};\ a_4 = E_{KL \cdot PTI}.$$

We shall primarily concern ourselves with the period 1923-39, using the nominal average price P'_N (table, page 19, col. 1) as a price series, and the estimate ot nominal income I_N (the real value is given in table, page 19, col. 7) as an income series. One gets the following coefficients of correlation between the variables.

Coefficients of correlation (total).

	log P'_N	log T	log I_N	log L
log K	— 0.88	0.71	— 0.16	0.71
log P'_N		— 0.53	0.40	— 0.32
log T			— 0.52	0.74
log I_N				0.13

The coefficient of correlation between price and quantity is negative and numerically greatest, while the coefficient of correlation between quantity and income is also negative, but insignificant. The table below gives the regression coefficients obtained by successively introducing the independent variables and the coefficient of correlation, R, between the observed values of log K and the values obtained from the regression equation.

On the other hand, the standard error for the regression coefficients is not included in the tables. Owing to a dependency that seems to exist between the successive observations in the time series, the significance of these standard errors is not the one desired. To give an idea of the size of conventional calculated standard errors of this type, however, examples will be given later for one of the calculated regression equations.

Coefficients of regression.

1		$\log P'_N$	$\log T$	$\log I_N$	$\log L$	R
1	$\log K = a_0 +$	$- 0.423$				0.881
2	$\log K = a_0 +$	$- 0.336$	0.457	—	—	0.926
3	$\log K = a_0 +$	$- 0.467$	—	0.173	—	0.906
4	$\log K = a_0 +$	$- 0.353$		—	0.711	0.977
5	$\log K = a_0 +$	$- 0.372$	0.705	0.316	—	0 992
6	$\log K = a_0 +$	$- 0.373$	0.868	0.374	0.173	0.993

lary value) by indirectly introducing the variable L in the lary value) by indirectly introducting the variable L in the regression analysis. This is done by using the prices and incomes divided by the index coefficient L, that is to say, the real prices and incomes, as variables.

It is common to justify the introduction of real prices and incomes by, among other things, the argument that if all prices and incomes are changed in the same degree, then the demand for a certain commodity will remain unaltered.

Technically speaking, this transformation means that S is minimized, providing that

(1) $$a_1 + a_3 + a_4 = 0 .$$

In fact, the sum of squares, S, can also be written as follows:

$$S = \sum_{v=1}^{n} \left(\log K_v - a_0 - a_1 \log \frac{P_v}{L_v} - a_2 \log T_v - a_3 \log \frac{I_v}{L_v} - (a_1 + a_3 + a_4) \log L_v \right)^2 .$$

Only if the elasticities fulfill condition (1) can the effect of the changes in the cost of living on the quantity demanded be eliminated by using real price and income values as independent variables in regression analysis.

In the particular case where the negative nominal price elasticity is numerically equal to the positive nominal income elasticity, the elasticity with regard to the index coefficient L would have to be set at zero.

A minimizing of S means a maximizing of the coefficient of correlation, R. If one now introduces a condition during this maximizing, then the maximum value obtained must be less than, or at least equal to, the maximum value obtained when the condition in question does not require to be fulfilled.

The coefficient of correlation R which belongs to a relationship with real prices and incomes is thus always less than, or equal to, the coefficient of the corresponding relationship with nominal prices and incomes and the index coefficient L as variables.

The following coefficients of correlation and regression were obtained for the real price and income series.

Coefficients of correlation.

	$\log P'_R$	$\log T$	$\log I_R$
$\log K$	— 0.96	0.71	— 0.46
$\log P'_R$		— 0.67	0.55
$\log T$			— 0.83

Coefficients of regression.

II		$\log P'_R$	$\log T$	$\log I_R$	R
1	$\log K = a_0 +$	— 0.406	—	—	0.959
2	$\log K = a_0 +$	— 0.369	0.174	—	0.964
3	$\log K = a_0 +$	— 0.369	0.651	0.300	0.992

If the two alternative price series, P'' and P''' respectively, are used, one gets:

Coefficients of regression.

III		$\log P''_R$	$\log P'''_R$	$\log T$	$\log I_R$	$\log R$
1	$\log K = a_0 +$	— 0.376	—	—	—	0.949
2	$\log K = a_0 +$	— 0.320	—	0.782	0.368	0.995
3	$\log K = a_0 +$	—	— 0.282	—	—	0.964
4	$\log K = a_0 +$	—	— 0.274	0.531	0.304	0.993

The above relationship has been worked out from figures covering the period 1923-39. From the values of the independent variables for 1939 and 1940, the average purchases during these years have been worked out for certain of the above relationships. As a result of the great increase in the price of liquor in November 1939, there was a marked decrease in the average consumption, from 20.9 (1939) to 16.9 (1940) litres per year and ration book holder.

Year	Average purchase	Average purchase, calculated from relationship		
		I. 6	II. 3	III. 4
1939	20.91	21.28	21.40	21.42
1940	16.90	17.93	18.55	18.93

The fact that the agreement is not so good indicates that the price elasticity at the time of the increase in prices during 1939-40 was absolutely greater than at the time of the increase in prices at the beginning of the 1930's (cf. Chap. III, § 4).

Applying regression analysis to the period 1923-40, we get the following coefficients:

Coefficients of regression.

IV		$\log P'_R$	$\log P'''_R$	$\log T$	$\log I_R$	R
1	$\log K = a_0 +$	— 0.456	—	—	—	0.969
2	$\log K = a_0 +$	—	— 0.326	—	—	0.961
3	$\log K = a_0 +$	— 0.443	—	0.538	0.286	0.982
4	$\log K = a_0 +$	—	— 0.340	0.338	0.271	0.973

The average purchases during 1939 and 1940 calculated from the above relationships are as follows:

Year	Average purchase	Average purchase, calculated from relationship	
		IV. 3	IV. 4
1939	20.91	21.03	20.93
1940	16.90	17.83	18.12

As an illustration of the fit, the following figures for average purchases have been worked out from relationship II.3. The coefficient with standard errors worked out in the usual way[1] are:

$$a_0 = 0.0217;$$

$$E_{KP'_R \cdot TI_R} = 0.369 \pm 0.019;$$

$$E_{KT \cdot P'_R I_R} = 0.651 \pm 0.094;$$

$$E_{KI_R \cdot P'_R T} = 0.300 \pm 0.044.$$

The calculated and observed values for the average consumption per ration book holder in litres per year.

Year	Calculated according to II. 3	Observed
1923	23.27	23.37
24	24.91	24.91
25	25.11	25.21
26	25.10	24.92
27	24.81	24.75
28	24.63	24.72
29	24.90	24.69
30	24.82	24.65
31	24.47	24.59
32	22.15	22.92
33	20.41	20.06
34	20.69	20.76
35	21.15	21.23
36	21.38	21.59
37	21.82	21.97
38	22.12	22.45
39	21.40	20.91

[1] See for instance: Whittaker-Robinson, »The Calculus of Observations». London-Glasgow 1944, Chap. IX.

H. Cramér, »Mathematical Methods of Statistics», Princeton, 1946, § 29.12.

A corresponding analysis of the annual expenditures, say O, gives the following elasticity coefficients:

Coefficients of regression.

Period		log P'_R	log P''_R	log P'''_R	log T	log I_R
23-39	log $O = a_0 +$	0.631	—	—	0.651	0.300
23-40	log $O = a_0 +$	0.557	—	—	0.538	0.286
23-38	log $O = a_0 +$	—	0.586	—	0.666	0.355
23-39	log $O = a_0 +$	—	—	0.465	0.844	0.293
23-40	log $O = a_0 +$	—	—	0.439	0.834	0.304

Since

$$\log O = \log K P' = \log K + \log P',$$

one has the following formal relationships between the quantity and expenditure elasticities:

$$E_{OP' \cdot TI} - E_{KP' \cdot TI} = 1; \quad E_{OI \cdot P'T} = E_{KI \cdot P'T}; \quad E_{OT \cdot P'I} = E_{KT \cdot P'I}.$$

For another price series, such as P'', one has

$$E_{OP'' \cdot TI} - E_{KP'' \cdot TI} = E_{P'P'' \cdot TI}; \quad E_{OI \cdot P''T} - E_{KI \cdot P''T} = E_{P'I \cdot P''T};$$

$$E_{OT \cdot P''I} - E_{KT \cdot P''I} = E_{P'T \cdot P''I}.$$

These relationships follow immediately from the mathematical definition of a partial elasticity. By comparing the moment matrices one finds that the relationships are also valid for the corresponding coefficients of regression.

If $E_{P'P''} < 1$ then $E_{OP''} - E_{KP''} < E_{OP'} - E_{KP'} = 1$.

The regression analysis shows that (secondary subscripts omitted)

$$|E_{KP'}| > |E_{KP''}|.$$

This result is an expression of the fact that the changes in the weighted average price, P'', are greater than the changes in the unweighted price, P', which is an illustration of the price trend, dampened down by the changes in consumption.

Further, one has

$$|E_{KP''}| > |E_{KP'''}|.$$

This inequality is also a result of the changes in consumption, if one assumes that when a price-change takes place, the prices of the different qualities of liquor are changed in the same ratio as the price of 40 % distilled spirits or P''.

However, as it is more common for the price of all qualities of liquor to be raised by the same amount, the relative increase in the price of 40 % distilled spirits will be greater than the average increase in price. In such case, even apart from the shift in consumption, one can expect the same relationship between price elasticities as that mentioned above.

Corresponding relationships are obtained for the expenditure elasticities, that is to say,

$$E_{OP'} > E_{OP''} > E_{OP'''}.$$

If we neglect the elasticity of quantity with regard to size of ration (the ration elasticity), which will be dealt with later, the results of the regression analysis can be summed up as follows. During the periods 1921-31 and 1934-38, the real price series for liquor have been relatively stable. The price-changes during the period 1929-39 are chiefly concentrated to 1932 and 1933. The consumption trend during these periods of relatively stable prices has been the determining factor for the regression coefficient of the income variable. The decreased consumption during 1932 and 1933 has been assumed to have been caused by increases in prices and decreased incomes. The effect of the decreases in income is determined by means of the regression coefficient of the income variable which is determined by the consumption trend during periods of stable prices. That part of the decrease in consumption during 1932 and 1933 which

is not explained in this way by the change in income over the same period is ascribed to price changes, and thus determines the regression coefficient of the price variable, that is to say, the price elasticity. The relative increase in price during this period is 4 to 5 times greater than the relative change in income during the same period. It follows from this dominance of the price trend that if one had obtained another value for the income elasticity, this would have brought about a change in the price elasticity 4 to 5 times smaller than the difference between the income elasticities. A slight misjudgement of the income elasticity is thus of little significance for the price elasticity.

The result of a simplified analysis of this kind is given in the table below. In this case, the purchasing frequency — that is to say, the ratio between the quantity purchased and the corresponding average ration — has been taken as the dependent variable (cf. Chap. II, § 8).

Coefficients obtained by simplified regression analysis.

Line	Period	Price	Income	E_{KP}	E_{KI}
1	1927-38 except 1932	P'_R	I_R	— 0.33	0.43
2	1927-38 except 1932	1927-31: 2.16 1933-38: 3.17	I_R	— 0.34	0.48
3	1927-31	2.16	I_R	—	0.43
4	1933-38	3.17	I_R	—	0.52

Line No. 1 shows the regression coefficients for the frequency of purchasing during the period 1927-38, excluding 1932. The independent variables are the average price P'_R and the income series I_R. Neglecting the very small price-changes during the periods 1927-31 and 1933-38, one gets the regression coefficients in line No. 2. A separate determination of the income elasticities gives the values in lines 3 and 4.

§ 5. Regression analysis of the quantities of wine sold by bottle stores

During the period 1927-38, the prices of wines sold by bottle stores were relatively stable. However, as the prices of spirits were raised during this same period, a study of wine sales during 1927-38 may be expected to give a certain amount of information regarding the change in wine sales in relation to change in income and change in the price of liquor. One can begin by carrying out a regression analysis using the annual amount of wine sold as the dependent variable, and an income series and a price series for liquor as the independent variables. To eliminate the effect of the increasing number of ration book holders on the total quantity of wine sold annually, we shall take the annual quantity of wine sold per ration book holder as the dependent variable. By this means a stratification effect[1] can be introduced. For, if the number of ration book holders is increased by persons who, let us suppose, consume less wine than the others, this fact alone will mean a decreased consumption of wine per ration book holder, even when prices and incomes are stable. As most of the price changes have taken place over a relatively short period, the calculated price elasticity will, as pointed out elsewhere, be a so called short period elasticity. In such case, the direct result of the stratification effect in question will be of less importance in determining the price elasticity than in determining the income elasticity. A stratification effect of the type described above would, in the case of increased incomes, produce an income elasticity that was too low. If the new ration book holders bought more wine than the others, the increased incomes would result in the income elasticity being too high. It is possible to take account of this fact during regression analysis by taking the annual amount of wine sold as the dependent variable, and introducing the number of ration book holders as one of the independent variables. The situation is analogous to that discussed in chapter II, § 8. In the first case dealt with, the elasticity of wine sales with regard to the number of ration book holders would be less than 1, in the second case greater than 1.

[1] Wold 1940, § 23.

As an alternative, the amount of wine sold can be set in relation to the spirits ration. Assuming that the ratio of wine purchases to the ration for spirits is the same for each ration group, then the stratification effect referred to above will be eliminated.

Another factor tends to make the calculation of the income elasticity difficult. It has been said, in reference to income elasticity for spirits, that this elasticity is mainly determined by the trend in consumption during the two periods when the price was stable. The same thing holds in the case of income elasticity for wine sold through bottle stores. The increase in the amount of wine sold, set in relation to the number of ration book holders or the total liquor ration, may conceivably be explained to a greater or minor degree by other factors besides income trend, and the increase may have been apparent even if the incomes had remained unchanged.

The results are as follows:

Coefficients of regression.

	$\log P'_R$	$\log P'''_R$	$\log I^T_R$	R
$\log V_1 = a_0 +$	— 1.008	—	1.266	0.993
$\log V_1 = a_0 +$	—	— 0.716	1.323	0.990
$\log V_2 = a_0 +$	—	— 0.641	1.498	0.983

V_1 = quantity of wine sold per ration book holder.

V_2 = quantity of wine sold per litre ration of liquor.

As the average price of wine remained practically constant, the elasticity for expenditure on wine agrees with the corresponding quantity elasticity.

It may once more be pointed out that the price elasticities are elasticities for wine consumption with regard to the price of spirits.

Using a simplified regression analysis (Chap. I, § 4) for determining income elasticity, one gets

$$1927\text{-}31: \quad E_{V_1 I} = 1.28; \quad E_{V_2 I} = 1.71.$$

$$1933\text{-}38: \quad E_{V_1 I} = 1.44; \quad E_{V_2 I} = 1.55.$$

§ 6. The relationship between income elasticities for two commodities

In determining the elasticity for the purchases of a commodity with regard to the total income of the consumer, one uses a series which is presumed to be an expression of the changes in this income. If the income series used is not a satisfactory expression of these changes, the calculated elasticity will lose its original significance. It should be pointed out that the absolute size of the income is of secondary importance, insofar as a constant elasticity is determined only by the relative changes in the total income. If it is felt that these changes cannot be estimated one can, under certain conditions stated below, calculate the relationship between the income elasticities for purchases of two commodities.

Let K_1 and K_2 respectively represent the purchases of two commodities during a unit of time. The corresponding income elasticities will be $E_{K_1 I}$ and $E_{K_2 I}$, where I is the total income of the consumers in question. Further, let η_1 and η_2 be two variables, which we shall assume are uncorrelated with the logarithm for income. Take

$$\begin{aligned} \log K_1 &= E_{K_1 I} \cdot \log I + \eta_1 ; \\ \log K_2 &= E_{K_2 I} \cdot \log I + \eta_2 . \end{aligned} \tag{1}$$

The log values of the variables are assumed to be given as deviations from their average values.

On account of the condition laid down for the correlations between the variables one gets the following dispersion squares for the logs of the purchases:

$$\begin{aligned} \Sigma \log^2 K_1 &= E^2_{K_1 I} \cdot \Sigma \log^2 I + \Sigma \eta_1^2 ; \\ \Sigma \log^2 K_2 &= E^2_{K_2 I} \cdot \Sigma \log^2 I + \Sigma \eta_2^2 , \end{aligned} \tag{2}$$

or

$$\frac{E_{K_1 I}}{E_{K_2 I}} = \sqrt{\frac{\Sigma \log^2 K_1 - \Sigma \eta_1^2}{\Sigma \log^2 K_2 - \Sigma \eta_2^2}} . \tag{3}$$

The difficulty about using this expression is obviously in determining the dispersions for the variables η_1 and η_2.

If one introduces the condition

$$\frac{\Sigma\, \eta_1^2}{\Sigma \log^2 K_1} \quad \frac{\Sigma\, \eta_2^2}{\Sigma \log^2 K_2}, \tag{4}$$

one gets

$$\frac{E_{K_1 I}}{E_{K_2 I}} = \sqrt{\frac{\Sigma \log^2 K_1}{\Sigma \log^2 K_2}}. \tag{5}$$

Equation (5) can be written in the form of elasticities as follows:

$$\frac{E_{K_1 I}}{E_{K_2 I}} = \sqrt{\frac{E_{K_1 K_2}}{E_{K_2 K_1}}}. \tag{6}$$

Condition (4) means that the coefficient of correlation between the logarithm of the quantity and the logarithm of the total income assumed to be equally great for both commodities. One can express the same thing by saying that the variations in income »explain» just as much of the variations in both quantity series.

If the changes in income can explain a smaller part of the variations in the series K_1 than in the series K_2, then the quotient of the income elasticities is less than that given in equation (5) and vice versa.

It is worth mentioning, incidentally, that if we regard a diagram showing the logarithmic values of the quantities, equation (5) shows that the relationship between income elasticities is determined by the gradient of the so called diagonal regression line.

Further, it should be noted that if η_1 and η_2 are uncorrelated, one easily obtains

$$E_{K_1 K_2} < \frac{E_{K_1 I}}{E_{K_2 I}} < \frac{1}{E_{K_2 K_1}}.$$

This assumption regarding correlation would, however, appear to be unrealistic.

To give a numerical illustration of equation (5), let us look at the annual values for the quantity of wine sold per ration book holder, (V_1), and the purchasing frequency for spirits,

(U), during the intervals 1927-31 and 1933-38. During each of these intervals the prices of wine and spirits have been relatively stable.

For the period 1927-31, one gets

$$E_{V_1 U} = 3.152; \qquad \frac{1}{E_{U V_1}} = 3.516,$$

and as a determination of the relationship between income elasticities

$$\sqrt{\frac{E_{V_1 U}}{E_{U V_1}}} = 3.329.$$

For the period 1933-38, one has

$$E_{V_1 U} = 2.791; \qquad \frac{1}{E_{U V_1}} = 3.009; \qquad \sqrt{\frac{E_{V_1 U}}{E_{U V_1}}} = 2.898.$$

If the real income according to communal taxation returns is taken as the income variable, one gets the following income elasticities

$$1927\text{-}31: \; E_{V_1 I} = 1.281; \quad E_{UI} = 0.384.$$

$$1933\text{-}38: \; E_{V_1 I} = 1.444; \quad E_{UI} = 0.504.$$

The relations between these income elasticities for amount of wine purchased and the frequency of purchasing are thus

3.33 and 2.86 respectively.

For the relationship between income elasticities for the variables V_2 — that is to say the amount of wine purchased per litre ration of spirits — and U, one gets, using the above method, the following:

1927-31: 1.38; 1933-38: 3.09.

CHAPTER II

This chapter is devoted to an examination of what is meant by demand elasticities for a rationed commodity. In making this examination, it is assumed that the rationing is such that a consumer is permitted to purchase a maximum quantity A, hereafter referred to as the ration or the size of the ration, during a given rationing period.

Suppose all consumers of a commodity increase their purchases when the price is decreased. If the commodity is rationed, then (apart from transfers of rations) only those consumers who not take out their full rations will be able to increase their purchases. Hence, since only a certain proportion of the consumers react to a change in price, the price elasticity of the demand for the rationed commodity should be numerically smaller than the price elasticity for the commodity when it is unrationed.

Further, one can expect the elasticity of the rationed commodity to be numerically smaller, the greater the number of consumers there are who take out their full rations. If all consumers take out their full rations, there can be no increase in the size of purchases when the price is lowered. Hence, in such case, the price elasticity will be equal to zero.

The elasticity for the rationed commodity is thus dependent on the distribution of consumption and also on the size of the ration.

Let us suppose that the elasticity of demand is constant and of the same magnitude for each consumer who purchases a quantity that is less than the size of his ration. This elasticity will then agree with the elasticity for the total demand of the commodity in question when this commodity is unrationed.

In § 1, the relationship between this elasticity and the elasticity of the total demand for the rationed commodity is taken under consideration. It is found that the quotient of the elasticity for the total demand of the rationed commodity, and the individual elasticity, is equal to the quotient of the consumption of those consumers who do not take out their full rations, and the total consumption.

Let E'_{KP} be the price elasticity of the demand for the rationed commodity, E_{KP} the individual elasticity and E_{KA} the elasticity of the demand for the rationed commodity with regard to the size of the ration or, more simply, the ration elasticity.

Further, let F denote the relative number of consumers who take out their full rations, and U denote the purchasing frequency, that is to say, the ratio between the average amount purchased and the size of the ration.

One gets the following fundamental relationship:

$$E_{KA} = \frac{F}{U};$$

$$E'_{KP} = E_{KP}(1 - E_{KA}).$$

If one knows two of these elasticities, it is thus possible to work out the third.

The elasticity relationships obtained in § 1 are founded on the assumption that rationing affects only the purchases of the consumers who, in the absence of rationing, would have purchased a quantity greater than the size of their rations, during the rationing period. These consumers are assumed to take out their full rations.

§ 1. is ended with a few remarks on the case where rationing, even in other respects, causes changes in the quantities purchased. However, to classify the complications produced by transfers of rations would require a detailed knowledge of the nature of these transfers.

In § 4, the purchases are treated as stochastic variables. If the distribution function for the purchases made by a person during a rationing period fulfills certain conditions, one will obtain relationships analagous to those in § 1.

In § 2, certain relationships between the elasticity of the demand for a rationed commodity and the distribution of consumption are dealt with for the case where the distribution is logarithmico-normal. If the individual elasticity is constant, it is shown that the numerical value of the price elasticity of the demand for the rationed commodity is a decreasing function of both the purchasing frequency and the dispersion of the distribution of consumption.

The case where the individual elasticities are assumed to vary is dealt with in § 3. Here too, some consequences and criteria of this variation are given.

In § 5, the relationship between the elasticity of the total demand for a commodity with regard to the total incomes of consumers (market elasticity) and the individual income elasticities is taken under consideration.

If these individual income elasticities are equal, the individual elasticity will agree only in certain cases with the market elasticity. Under certain conditions, implying a levelling out of incomes, the market elasticity will lie closer to the value 1 than the individual elasticity.

In § 5 it is shown that the market income elasticity of the demand for a rationed commodity is equal to the product of the following three factors:

1) The market elasticity for those consumers who do not take out their full ration.
2) The ratio between the purchases of these consumers and the total consumption.
3) The elasticity for the incomes of these consumers with regard to the total income.

Finally, consideration is given to the case where the size of the ration varies with the size of income. The reasoning here is linked up to the results obtained in this respect in chapter IV.

Suppose that the rationing regulations are interpreted in such a way that the purchasing frequency will be the same for all income groups. In that case the market income elasticity of the demand for the rationed commodity will agree with both the market elasticity of those consumers who do not take out their full rations and the elasticity of the average size of the

ration with regard to income. The relationships obtained in this connection are of importance in judging the income elasticities calculated from statistical material of the kind dealt with in chapter IV.

We have seen that the relationship between the individual elasticities and the elasticity of the demand for the rationed commodity is dependent on the values F and U defined above.

Changes in price, income and size of rations, produce changes in the variables F and U. The relationship between the elasticities mentioned above is a function only of the purchasing frequency U if each value of U corresponds to one and the same value of F, irrespective of the price level, the size of income or the size of the ration. This one-valued relationship holds for the case where the individual price and income elasticities are equal and where, also, it is assumed that the incomes are changed in the same ratio.

A knowledge of the one-valued relationship in a case such as this means that the ratio between the individual elasticity and the market elasticity of the demand for the rationed commodity can be calculated for a certain purchasing frequency. Then, if one knows the market elasticity at this purchasing frequency, it is possible finally to calculate the individual elasticity.

According to § 6, the relationship between F and U may also be used for calculating the effect of a change in rationing.

In § 7 the elasticity relationships in the case where rationing is differentiated are taken under consideration.

In § 8, the meaning of the elasticity of demand for a rationed commodity with respect to the average ration is examined. In making this examination, it is assumed that the commodity is differentially rationed. When the rationing regulations are interpreted in such a way that the purchasing frequencies in the different rationing classes will be equal and constant, the elasticity in question amounts to 1.

The conditional regression analysis carried out in this paragraph shows that the values obtained for price elasticity of the demand for liquor are, practically speaking, independent of the assumptions made regarding the interpretation of rationing regulations.

§ 1. Price elasticity of the demand for a rationed commodity when the individual elasticities are equal

A. Let us consider the consumption of a commodity which is rationed in such a way that a consumer is entitled to purchase a maximum quantity A during a ration period. We shall call A the ration or the size of the ration. Further, we shall assume that the rationing with which we are dealing does not affect the quantities purchased by those consumers who would have bought a quantity less than or equal to A had there been no rationing, and also that the remaining consumers purchase quantity A within the rationing period. If all the consumers purchase quantity $A_0(P) < A$ when the price is P, then in the case of rationing, the price elasticity for the consumption of the rationed commodity will agree with the price elasticity for consumption when the commodity is unrationed. When $A_0(P) > A$, the price elasticity for the rationed commodity will be zero. In extreme cases such as these, the interpretation of the price elasticity for the rationed commodity is obvious.

Now suppose the quantities consumed are unequal, and let $F(X, P)$ denote the relative number of consumers who buy a quantity equal to, or greater than, X, when the price is P. When the commodity is rationed, $F(A, P)$ will thus denote the number of consumers who purchase the full ration when the price is P.

This distribution of consumption can now be interpreted in different ways. Let the quantity purchased by the consumer during the ration period in question be expressed by $m_i + x_i$ where m_i is the average quantity purchased during a ration period, and x_i are independent stochastic variables with a mean value $= 0$.

If all m_i's are equal, the distribution of consumption will be stochastic. If the consumers are ranked according to the size of their purchases, the coefficient of correlation between the rank numbers for two ration periods will amount to zero, providing conditions remain unchanged.

If all the x_i's amount to zero, the coefficient of correlation between the rank numbers will amount to 1, and the distribution of consumption will be of a non-stochastic nature.

Finally, the distribution may be interpreted as a combination of these cases.

B. We shall deal firstly with the case where the distribution function is of a non-stochastic nature, and begin by considering the following problem.

Suppose that the price elasticity for the demand of an unrationed commodity is equal to E for each consumer. The commodity is then rationed; the quantity purchased at price P_0 is K_0, and the number of N consumers who purchase their full ration is NF_0. Following a reduction in price from P_0 to P_1, one gets the values K_1 and NF_1, respectively. How big is the elasticity E?

The quantity K_0 is composed of two parts, namely the quantity purchased by those persons who take out the full ration, and the quantity purchased by the remainder, say k_0. Thus

$$K_0 = k_0 + ANF_0.$$

Quantity K_1 can be split up in the same way

$$K_1 = k_1 + ANF_1.$$

We are assuming now that the price elasticity is negative, that is, a reduction in price will result in increased purchasing. In such case, F_1 is greater than F_0. Then the quantity k_0 may also be split up into two quantities. Let k_0' be the quantity purchased by those consumers who even in the case of the lower price P_1 do not take out their full rations, and let k_0'' be the quantity purchased by the $N(F_1 - F_0)$ consumers who do not take out their full ration when the price is P_0, but who consume quantity A when the price is P_1. We then get

$$\begin{cases} K_0 = k_0' + k_0'' + ANF_0; \\ K_1 = k_1 + NA(F_1 - F_0) + ANF_0. \end{cases}$$

From this it follows that all one knows about NF_0 persons is that they take out their full ration when the price is P_0 and consequently when it is P_1 also. Their reactions to price changes in the price interval in question has, in the case

where the commodity is unrationed, no effect on the quantity of the rationed commodity they purchase in this interval. For $(N - NF_1)$ consumers, on the other hand, rationing under the conditions stipulated has not had any effect on the quantities purchased. The determination of the elasticity sought for, E, must therefore be carried out on the basis of the reaction of these consumers to a change in price. To determine the quan-

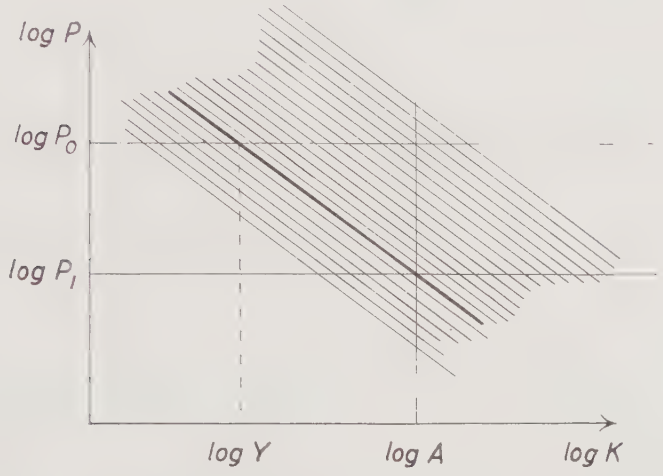

Fig 1

tity they purchase at price P_0, it is necessary to calculate the quantity k''_0. This quantity is governed not only by the conditions of elasticity, but also by the distribution of purchases made by the $N(F_1 - F_0)$ consumers in question. At price P_0, these consumers must have bought a quantity which, at the most, is equal to A. The smallest quantity, let us say Y, will have been purchased at this price by the person who at price P_1 would purchase quantity A, even if the commodity in question were unrationed (cf. fig. 1). Since the individual price elasticity for this person is E, one gets

$$\frac{\log A - \log Y}{\log P_1 - \log P_0} = E \text{ or } Y = A \left(\frac{P_0}{P_1}\right)^E.$$

Let us now suppose that the purchases in question are uniformly distributed over the interval Y to A. One then has

$$k_0'' = N(F_1 - F_0)\,\frac{A}{2}\left[1 + \left(\frac{P_0}{P_1}\right)^E\right].$$

Since the elasticity for the quantities purchased by the $(N - NF_1)$ consumers, whose purchases are unaffected by rationing must be E, one gets

$$\left(\frac{P_0}{P_1}\right)^E = \frac{k_0'}{k_1} = \frac{K_0 - N(F_1 - F_0)\,\frac{A}{2}\left[1 + \left(\frac{P_0}{P_1}\right)^E\right] - ANF_0}{K_1 - ANF_1},$$

or finally,

$$(1) \qquad E = \frac{\log\left(K_1 - AN\,\frac{F_0 + F_1}{2}\right) - \log\left(K_0 - AN\,\frac{F_0 + F_1}{2}\right)}{\log P_1 - \log P_0}.$$

It should be pointed out that the same expression is naturally valid even in the case of an increase in price, or when the elasticity E is positive.

Expression (1) is derived on the basis of a special assumption regarding the distribution of consumption for certain consumers. If one allows this distribution of consumption to be arbitrary, then obviously

$$N(F_1 - F_0)\cdot A\left(\frac{P_0}{P_1}\right)^E \leqq k_0'' \leqq N(F_1 - F_0)\,A,$$

that is to say

$$K_0 - ANF_0 - N(F_1 - F_0)\,A \leqq k_0' \leqq K_0 - ANF_0 - N(F_1 - F_0)\,A\left(\frac{P_0}{P_1}\right)^E,$$

and finally

$$\frac{K_0 - ANF_0 - N(F_1 - F_0)\,A}{K_1 - ANF_1} \leqq \left(\frac{P_0}{P_1}\right)^E = \frac{k_0'}{k_1} \leqq \frac{K_0 - ANF_0 - N(F_1 - F_0)\,A\left(\frac{P_0}{P_1}\right)^E}{K_1 - ANF_1}.$$

After transforming one gets, when $E < 0$

$$(2)\qquad \frac{\log(K_1 - ANF_0) - \log(K_0 - ANF_0)}{\log P_1 - \log P_0} \geqq E \geqq \frac{\log(K_1 - ANF_1) - \log(K_0 - ANF_1)}{\log P_1 - \log P_0}.$$

Now let the change in price be infinitesimally small and take

$$P_1 = P_0 + dP; \quad K_1 = K_0 + dK; \quad F_1 = F_0 + dF.$$

From (2) one then gets

$$\frac{\log\left(1 + \frac{dK}{K_0 - AF_0N}\right)}{\log\left(1 + \frac{dP}{P_0}\right)} \geqq E \geqq \frac{\log\left(1 + \frac{dK}{K_0 - (AF_0 + A\,dF)N}\right)}{\log\left(1 + \frac{dP}{P_0}\right)}.$$

Let $dK,\ dF \to 0$ when $dP \to 0$.

One then gets

$$(3)\qquad \lim_{dP \to 0} \frac{dK_0}{K_0}\frac{P_0}{dP_0} = E\left(\frac{K_0 - AF_0N}{K_0}\right).$$

The term on the left gives the elasticity for the total consumption of the rationed commodity and will henceforth be denoted by E^r. Equality (3) tells us this: The price elasticity for the total consumption of a rationed commodity is equal to the individual elasticity multiplied by the ratio between the consumption of those consumers who do not consume their full ration, and the total consumption.

Returning now to the example chosen, let us investigate the significance of the elasticity given by the familiar relation,

$$E^r = \frac{\log K_1 - \log K_0}{\log P_1 - \log P_0}.$$

Regarding F as a function of the price, P,

$$F(P_0) = F_0; \quad F(P_1) = F_1,$$

we get from (3)

$$\log K_1 - \log K_0 = \int_{P_0}^{P_1} \frac{dK}{K} = E \int_{P_0}^{P_1} \frac{K - ANF}{K} \frac{dP}{P},$$

or

$$E' = E \frac{\int_{P_0}^{P_1} \frac{K - ANF}{K} \frac{dP}{P}}{\int_{P_0}^{P_1} \frac{dP}{P}} \tag{4}$$

By means of (4), the ratio between the elasticity for the rationed commodity and the individual elasticity is represented as a weighted average by the ratio between the amount purchased by those consumers who take out their full ration, and the total amount purchased. The weights are the inverted price.

Introducing the term U for the purchasing frequency, that is to say, the ratio between the quantity purchased and the maximum quantity that rationing entitles the consumers to purchase, relation (3) can be rewritten in the form

$$E' = E\left(1 - \frac{F}{U}\right). \tag{5}$$

One sees immediately that in the case where $E < 0$ one gets,

$$0 \geq E' \geq E.$$

In other words, the numerical value of the elasticity for the rationed commodity will be less than the numerical value of the individual elasticity, providing some consumers purchase their full ration.

The ratio, included in (5), between the relative number of consumers, F, who take out their full ration, and the purchasing frequency, U, is capable of special interpretation.

The total quantity of the rationed commodity purchased can be expressed by

$$K = -N \int_0^A X \frac{\delta F}{\delta X} dX + ANF = N \int_0^A F dX. \tag{6}$$

We can now introduce the term ration elasticity, that is to say the elasticity of the consumption with regard to the size of the ration.

This elasticity is obtained from (6).

Thus

$$E_{KA} = \frac{\delta K}{\delta A} \frac{A}{K} = \frac{F}{U}. \tag{7}$$

The ration elasticity is thus the same as the ratio between the relative number of consumers who take out their full ration, and the purchasing frequency.

Equation (5) may then also be written

$$E'_{KP} = E_{KP}\left[1 - E_{KA}\right]. \tag{8}$$

C. We have previously examined the meaning of elasticity of demand in the case of a rationed commodity. In doing so, it was assumed that rationing affected only the purchases of those consumers who, in the absence of rationing, would have bought larger quantities of the commodity in question than is possible when it is rationed. It was assumed that these consumers take out their rations in full.

Admittedly, this assumption is generally not realistic. The reactions of the consumers of a rationed commodity are likely to be affected by rations being transferred from one person to another, to a greater or lesser degree. The quantity, K, which a consumer purchases would thus consist of a quantity K_1 intended for personal consumption, and a quantity K_2 which

would be transferred to another person or persons. One then has the following relationship between the respective elasticities.

$$E = \frac{K_1 E_1 + K_2 E_2}{K}.$$

It is hard to make a comparison between the size of the elasticity for the quantity personally consumed and the size of the elasticity for the quantity transferred.

Suppose, for example, that in the absence of rationing the individual elasticities are of the same magnitude. Suppose also, that the effect of rationing is completely wiped out owing to those persons whose demand for the commodity in question is less than the size of their rations transferring their excess to the other consumers. The price elasticity for the quantity transferred will then be numerically greater than the individual elasticity providing, of course, that transfer does not take place to persons entitled to no ration at all. For the changes in the quantity consumed by the recipients of such transferred quantities must primarily be considered to take place in the quantity transferred. On the other hand, if the transfers are incomplete, that is to say, if there remains an unsatisfied demand on the part of those persons whose normal consumption is greater than the size of their rations, the ratio between the sizes of two elasticities may be the reverse. A more intensified system of transfer would thus appear to result in a numerically greater elasticity for the quantities transferred. However, to go into this more fully would necessitate assumptions regarding the distribution of the transferred quantities among the transferors.

The existence of transfers means that the distribution of consumption, F, must be regarded as a function of the size of the ration. One then has

$$K = N \int_0^A F(A, X, P)\, dX.$$

In calculating the ration elasticity, one has

$$E_{KA} = \frac{F}{U} + \frac{A}{K} \int_0^A \frac{\delta F}{\delta A} dX .$$

As the increased demand for transferred quantities produced by, let us say, a reduction in rations, would probably result in an increase in these quantities, one has, for the interval under consideration,

$$\frac{\delta F}{\delta A} \leqq 0 \quad \text{and} \quad E_{KA} < \frac{F}{U} .$$

Rationing may also conceivably affect personal consumption. For instance, a person whose normal consumption is slightly less than the size of his ration may, when rationing is introduced, increase his consumption up to the level of his ration. This would result in an increase in the numerical value of the price elasticity when an increase in prices takes place, with the result that the consumer, at the new price level, reverts to his original consumption. For similar reasons, this change in personal consumption produces a ration elasticity lower than the normal and can, in extreme cases, result in a negative ration elasticity.

§ 2. Illustration by means of a logarithmico-normal distribution of consumption

We shall now study a particular distribution of consumption and for this purpose we shall take the case where the logarithms of the quantities purchased at price P_0 are normally distributed, and have an average value $m(P_0)$ and a dispersion $\sigma(P_0)$.

Further, let us suppose that the consumers have the same function of elasticity which we shall denote by $E(P)$.

Thus, one has

$$\log K(P) = \log K(P_0) + \int_{P_0}^{P} \frac{E(Z)}{Z}\, dZ.$$

From this it follows that the logarithms of the quantities purchased at price P are also normally distributed, with

$$m(P) = m(P_0) + \int_{P_0}^{P} \frac{E(Z)}{Z}\, dZ; \quad \sigma(P) = \sigma(P_0) = \sigma.$$

The distribution of consumption is thus given by

$$(1) \qquad F(X,P) = \frac{1}{\sigma\sqrt{2\pi}} \int_{X}^{\infty} \frac{1}{Z} \exp -\frac{1}{2}\left(\frac{\log Z - m(P)}{\sigma}\right)^2 dZ.$$

We may now introduce the following terms for the normal distribution.

$$\Phi(X) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{X} \exp -\frac{1}{2} Z^2\, dZ; \quad \varphi(X) = \Phi'(X) = \frac{1}{\sqrt{2\pi}} \exp -\frac{1}{2} X^2,$$

and put

$$(2) \qquad h(X,P) = h = \frac{\log X - m(P)}{\sigma}.$$

The distribution of consumption can then be written as follows:

$$F(X,P) = 1 - \Phi[h(X,P)].$$

Let the average value and the dispersion of the quantities purchased at price P be $M(P)$ and $S(P)$ respectively. One then has the following relationship between the characteristics:[1]

$$M(P) = \frac{1}{\sigma\sqrt{2\pi}} \int_{0}^{\infty} \exp -\frac{1}{2}\left(\frac{\log Z - m(P)}{\sigma}\right)^2 dZ.$$

[1] C. E. Quensel, »On the logarithmico-normal distribution». Skandinavisk Aktuarietidskrift 1945, p. 141.

After the transformation of the variables

$$h(Z,P) - \sigma = Y.$$

one gets, finally

$$\log M(P) = m(P) + \frac{\sigma^2}{2}. \tag{3}$$

In a similar manner

$$\log S(P) = m(P) + \frac{\sigma^2}{2} + \frac{1}{2}\log(e^{\sigma^2} - 1). \tag{4}$$

Thus, (Cf. II, § 3, (4))

$$E_{M(P)P} = E_{S(P)P}.$$

If the commodity is rationed, with A denoting the size of the ration, one gets for the purchasing frequency $U(A,P)$

$$U(A,P) = \frac{1}{A}\int_0^A F(Z,P)\,dZ = \frac{1}{A}\int_0^A Z\frac{\delta F}{\delta Z}\,dZ + F(A,P).$$

After reducing, one has

$$F(A,P) = 1 - \Phi[h(A,P)]; \tag{5}$$

$$U(A,P) = \varphi(h)\cdot\frac{\Phi(h-\sigma)}{\varphi(h-\sigma)} + 1 - \Phi(h). \tag{6}$$

The following table shows the values obtained for $\sigma = 0.1$ and $\sigma = 1$.

h	F	$\sigma = 0.1$		$\sigma = 1$	
		U	$1 - \frac{F}{U} = \frac{E^r}{E}$	U	$1 - \frac{F}{U} = \frac{E^r}{E}$
− 2.0	0.023	0.822	0.972	0.210	0.892
− 1.5	0.067	0.862	0.922	0.321	0.792
− 1.0	0.159	0.901	0.824	0.462	0.657
− 0.5	0.309	0.935	0.670	0.617	0.500
0	0.500	0.962	0.480	0.762	0.343
+ 0.5	0.691	0.981	0.296	0.873	0.208
+ 1.0	0.841	0.992	0.152	0.943	0.108
+ 1.5	0.933	0.997	0.064	0.979	0.047
+ 2.0	0.977	0.999	0.022	0.994	0.017

With given values for the size of the ration, A, the purchasing frequency, U, and the relative number of persons, F, who take out their ration in full, one can calculate the characteristics m and σ and with them even M and S according to (3) and (4), from relations (5) and (6) by the aid of known values for the normal distribution.

We shall here point out some of the consequences of the properties of the normal distribution.

A. Let us first consider ration elasticity in the case where the size of the ration agrees with the average value $M(P)$ for the distribution of consumption.

Here

$$h(M(P), P) = \frac{\sigma}{2},$$

hence

$$\frac{F[M, P]}{U[M, P]} = \frac{1 - \Phi\left(\frac{\sigma}{2}\right)}{\Phi\left(-\frac{\sigma}{2}\right) + 1 - \Phi\left(\frac{\sigma}{2}\right)} = \frac{1}{2}.$$

This relationship tells us the following: If the size of the rations agrees with the average value for the distribution of consumption, the ration elasticity is equal to 0.5. From II, § 1, (8) we learn too that the individual price elasticity is, in such case, twice as great as the elasticity for the rationed commodity.

B. Either directly or by using the relationships

$$\frac{\varphi(x)}{\Phi(x)} > -x \text{ when } x \leqq 0; \qquad \frac{\varphi(x)}{1 - \Phi(x)} > x \text{ when } x \geqq 0;$$

one gets, interalia, the following inequalities

$$\frac{\delta F}{\delta \sigma} = 0; \quad \frac{\delta F}{\delta h} < 0; \quad \frac{\delta U}{\delta \sigma} < 0; \quad \frac{\delta U}{\delta h} < 0; \quad \frac{\delta \frac{F}{U}}{\delta \sigma} > 0; \quad \frac{\delta \frac{F}{U}}{\delta h} < 0.$$

Suppose now that the price is changed but the size of the ration remains constant. One then has

$$\frac{d\sigma}{dP} = 0; \quad \frac{dh}{dP} = -\frac{1}{\sigma}\frac{dm}{dP} - \frac{1}{\sigma}\frac{E}{P},$$

hence

$$\frac{dF}{dP} = \frac{\delta F}{\delta h}\frac{dh}{dP} < 0 \text{ when } E < 0. \tag{7}$$

When the price is increased, both the purchasing frequency and the number of persons who take out their full ration are reduced. Inequality (7) signifies that the ration elasticity decreases when the price is increased. At the same time the ratio between the price elasticity for the rationed commodity and the individual elasticity will increase. With individual elasticity constant, the numerical value of the price elasticity of the demand for the rationed commodity is thus an increasing function of the price and a decreasing function of the purchasing frequency.

If the size of the ration alone is varied, one has

$$\frac{dF}{dA} = \frac{\delta E}{\delta U}\cdot\frac{dU}{dA} = \frac{\delta F}{\delta h}\cdot\frac{1}{A\sigma} < 0.$$

Thus, the remarks concerning an increase in price apply also to an increase in the size of the ration.

Finally, we shall consider the case where the purchasing frequency and the mean value of the distribution of consumption are constant, but where the dispersion of the distribution of consumption (S) is varied. One then has

$$\frac{dU}{d\sigma} = \frac{\delta U}{\delta \sigma} + \frac{\delta U}{\delta h}\frac{dh}{d\sigma} = 0, \tag{8}$$

$$\frac{dF}{dS} = \frac{dF}{d\sigma}\cdot\frac{d\sigma}{dS} = \frac{\delta F}{\delta h}\cdot\frac{dh}{d\sigma}\cdot\frac{d\sigma}{dS}. \tag{9}$$

From (4) one gets

$$\frac{d\sigma}{dS} > 0$$

and hence from (8) and (9)

$$\frac{dF}{dS} > 0.$$

Expressing this in words, one can say that if the purchasing frequency, the size of the ration and the average value for the distribution of consumption are constant, the relative number of consumers who take out their full ration will vary with the dispersion of the distribution in such a way that this number will increase with the dispersion. From this it follows that the ratio between the price elasticity for the rationed commodity and the individual elasticity decreases when the distribution increases. The price elasticity of the rationed commodity is thus, in this case, a numerically decreasing function of the dispersion of the distribution of consumption.

§ 3. Price elasticity of a rationed commodity when the individual elasticities vary

A. We shall first examine the possibilities of obtaining an idea of the variation of the individual elasticities from the change in the distribution of consumption with the price.

1. It is clear that the distribution of consumption at price P, $F(X, P)$ is determined by the distribution function $F(X, P_0)$ and the function of individual elasticity, $E(X)$. On the other hand, the elasticity function is not determined by the distributions $F(X, P_0)$ and $F(X, P)$. One must, therefore, introduce special assumptions concerning the function $E(X)$.

Let $K(X_0, P)$ represent the demand function of a consumer who consumes quantity X_0 when the price is P_0, that is

$$K(X_0, P_0) = X_0.$$

The corresponding elasticity, regarded as a function of X_0 and P is defined by

$$E(X_0, P) = \frac{\delta K(X_0, P)}{\delta P} \frac{P}{K(X_0, P)}.$$

Expressing the demand function in the corresponding elasticity function, one gets

$$K(X_0, P) = X_0 \exp \int_{P_0}^{P} \frac{E(X_0, Z)}{Z} dZ. \tag{1}$$

Now let $K(X_1, P)$ and $K(X_2, P)$, where $X_1 < X_2$, be the demand functions for two consumers. Suppose the consumers are ranked according to the quantities each has bought.

Then, if the relationship

$$K(X_1, P) < K(X_2, P)$$

or

$$\frac{\delta K(X_0, P)}{\delta X_0} > 0,$$

holds for a certain price interval, $P_1 P_2$, the rank numbers of the consumers will remain unchanged for each price within the interval.

If this is so, we shall for the sake of brevity henceforth say that the supposition regarding consumption ranking is fulfilled in the price interval in question.[1]

The supposition regarding consumption ranking is always fulfilled when an increase in price takes place if the numerical value of the negative individual price elasticity is decreased when the quantity consumed increases. In the case of a reduction in price, the supposition is always fulfilled if the numerical value increases with the quantity purchased.

If the supposition regarding consumption ranking is fulfilled, one gets, from (1), the following condition for the elasticity function within the price interval:

$$X_0 \int_{P_0}^{P} \frac{\delta E}{\delta X_0} \frac{dZ}{Z} > -1.$$

[1] Cf. R. Frisch, »Notater til ökonomisk teori», Oslo 1946, exkurs 18 k.

If the elasticity function is independent of the price one gets

$$\frac{d\,E(X_0)}{d\,X_0} > -\frac{1}{X_0(\log P - \log P_0)} \qquad P_1 \leqq P_0, \quad P \leqq P_2.$$

The supposition regarding consumption ranking gives, for example, a definite significance to the elasticity of the median of the distribution of consumption. This elasticity is the elasticity of the individual who buys the median quantity. By comparing elasticities of quartiles or deciles, for example, it is possible to get an idea of the variation in the individual elasticity.

When the change in price is infinitesimally small, the assumption regarding consumption ranking is fulfilled. Letting X denote a moving coordinate, one gets

$$F(X, P) = F(X + d\,X, \; P + d\,P)$$

or

$$\frac{\delta\,F(X,P)}{\delta\,P} \cdot \frac{P}{F} = -\,E(X,P)\,\frac{\delta\,F(X,P)}{\delta\,X} \cdot \frac{X}{F}$$

or, simplifying the same thing,

$$E_{FP} = -\,E_{KP} \cdot E_{FX}. \tag{2}$$

Further, one has

$$\frac{\delta\,E_{FP}}{\delta\,P} \cdot P = \frac{\delta\,E_{FX}}{\delta\,X} \cdot X$$

and from (2) one gets the following relations between the elasticities of the distribution and the individual demand elasticities.

$$E_P(E_{FX}) = -\,E_{KP}\,E_X(E_{FP}) = -\,E_{KP}\,E_X(E_{FX}) + E_{KP}\,E_X(E_{KP}); \tag{3}$$

$$E_P(E_{FP}) = -\,E_{KP}\,E_X(E_{KP}) - E_{KP}\,E_X(E_{FX}) - E_P(E_{KP}),$$

denoting the elasticity of an elasticity by $E\,(E)$.

The relations (3) can be used in the following case. Suppose one knows the value of the distribution of consumption for two quantities, X_1 and X_2, at two price levels P_1 and P_2.

Let the individual elasticity E be constant throughout the interval and suppose further, that

$$E_X(E_{FP}) = E_1 .$$

It then follows, that

$$E_P(E_{FX}) = E_2 \text{ is constant.}$$

In the interval under consideration, one then has

$$\log F(X, P) = a_0 X^{E_1} P^{E_2} + a_1$$

where a_0, a_1, E_1 and E_2 are constants which may easily be expressed in the values given.

One finally obtains from the first equation in (3)

$$E_{RP} = -\frac{E_2}{E_1} .$$

2. Let the n th. moment, μ_n, for the distribution of consumption be defined by

$$\mu_n = \int_0^{\cdot} X^{n-1} F(X, P)\, dX .$$

For the elasticity of μ_n with regard to price, one gets

$$E_{\mu_n P} = \frac{\delta \mu_n}{\delta P} \cdot \frac{P}{\mu_n} = n \cdot \frac{\int_0^{\cdot} E(X) \left(-X^n \frac{\delta F}{\delta X}\right) dX}{\int_0^{\infty} -X^n \frac{\delta F}{\delta X}\, dX} . \tag{4}$$

Expressing the same thing in words: The elasticity of the n th. moment of the distribution of consumption divided by n is a weighted average of the individual elasticities. The weights

are the n th. power of the consumed quantities. The following special cases should be observed:

a) If the individual elasticities are equal, one has

$$\frac{1}{n} \cdot E_{\mu_n P} = E.$$

b) If the numerical value of the individual elasticity increases with the quantity consumed at price P_0, one gets

$$\frac{1}{n+1} \left| E_{\mu_{n+1} P} \right| > \frac{1}{n} \left| E_{\mu_n P} \right|.$$

c) When the numerical values of the elasticities decrease with the quantity, the reverse sign of inequality holds.

For the dispersion, σ, of the distribution of consumption in particular, one gets, when the elasticities are equal,

$$E_{\sigma P} = \frac{1}{2\sigma^2} \left[\mu_2 E_{\mu_2 P} - \mu_1^2 \cdot 2 E_{\mu_1 P} \right] = E.$$

Thus, the elasticity of the dispersion agrees in this case with the individual elasticity.

B. If, for a rationed commodity, the individual elasticities vary with the quantity purchased at price P_0, what then is the significance of the elasticity E obtained from II, § 1, (1)?

Using the same terms as before, one has, for an elasticity independent of price

$$Y = A \left(\frac{P_0}{P_1} \right)^{E(Y)}.$$

Concerning the elasticity conditions, we shall assume the following:

1) Every consumer who purchases less than quantity Y at price P_0 will purchase less than quantity A at price P.

2) Every consumer who purchases more than quantity Y at price P_0, will purchase quantity A at price P.

If these conditions are fulfilled for every A, then the supposition regarding consumption ranking is fulfilled.

Further, if the quantities purchased by the $N(F_1 - F_0)$ consumers are uniformly distributed over the interval $(Y; A)$, one gets

$$k_0'' = N(F_1 - F_0)\frac{A}{2}\left[1 + \left(\frac{P_0}{P_1}\right)^{E(Y)}\right].$$

Let the elasticity for the total amount purchased by the $N - NF_1$ persons whose purchases at prices P_0 and P_1 are not affected by the rationing of commodity be E_1, that is to say

$$\frac{k_0}{k_1} = \frac{K_0 - ANF_0 + F_1 - \frac{A}{2}N(F_1 - F_0)\left(\frac{P_0}{P_1}\right)^{E(Y)}}{K_1 - ANF_1} = \left(\frac{P_0}{P_1}\right)^{E_1}.$$

Now let us investigate the significance of formula II, § 1, (1) in the case of varying individual elasticities.

Thus, we may define an elasticity, E, as follows:

$$E = \frac{\log\left(K_0 - AN\frac{F_0+F_1}{2}\right) - \log\left(K_1 - AN\frac{F_0+F_1}{2}\right)}{\log P_0 - \log P_1}.$$

The elasticity E can be written as an average value of the elasticities $E(Y)$ and E_1 in the following manner

$$\left(\frac{P_0}{P_1}\right)^{E} = \frac{(K_1 - ANF_1)\left(\frac{P_0}{P_1}\right)^{E_1} + \frac{A}{2}N(F_1 - F_0)\left(\frac{P_0}{P_1}\right)^{E(Y)}}{K_1 - AN\frac{F_0+F_1}{2}}.$$

We shall now examine the case of an infinitesimally small change in price.

The quantity purchased, $K(P)$, of a rationed commodity can be written

$$K(P) = N\int_0^A F(X, P)\,dX.$$

For an infinitesimally small change in price, one gets, when introducing the elasticity E^r of the demand for the rationed commodity

$$E^r_{KP} = \frac{dK}{dP}\frac{P}{K} = \frac{N}{K}\int_0^A \frac{F(X,P)}{\delta P}\, P\, dX .$$

According to (2)

$$(5) \qquad E^r_{KP} = -\frac{N}{K}\int_0^A E(X)\frac{\delta F(X,P)}{\delta X}\, X\, dX =$$

$$= \frac{\int_0^A E(X)\, X\left(-\frac{\delta F}{\delta X}\right) dX}{\int_0^A X\left(-\frac{\delta F}{\delta X}\right) dX} \cdot \frac{K(P) - ANF(A,P)}{K(P)} .$$

The elasticity for the total consumption of a rationed commodity is equal to the product of a weighted average of the individual elasticities of those consumers who do not take out their full rations, and the ratio between the quantity purchased by these persons and the total quantity purchased. The quantities purchased are used as the weights in the weighted average.

Finally, let us consider the elasticity, $\overline{E^r_{KP}}$, for the rationed commodity given by

$$\overline{E^r_{KP}} = \frac{\log K_0 - \log K_1}{\log P_0 - \log P_1} .$$

Denoting the above weighted average of the individual elasticities by $M[E, P]$, one gets the following expression, corresponding to II. § 1, (4):

$$(6) \qquad \overline{E^r_{KP}} \quad M[E, P_\omega]\left[1 \quad \frac{\int_{P_0}^{P} E\, dP}{\int_{P_0}^{P} dP}\right] .$$

where P_ω is a price in the interval (P_0, P).

§ 4. Relations between elasticities when purchases are regarded as stochastic variables

When dealing with the distribution of consumption, it is natural to take into account the fact that the quantity bought should be regarded as a stochastic variable.

We shall assume, therefore, that the quantity purchased by a person during the ration period is a stochastic variable.

Let $\psi(X, Z, P)$ represent the probability that a person who on an average purchases quantity Z at price P buys a quantity equal to or less than X.

Let $F(X, P)$ represent the relative number of consumers who buy a quantity $\geqq X$. As before, let the distribution of the average quantities be $f(X, P)$ with

$$f(X, P) = \frac{\delta[1 - F(X, P)]}{\delta X} = -\frac{\delta F}{\delta X}.$$

One has, further,

$$(1) \qquad \overline{F(X, P)} = 1 - \int_0^\infty f(Z, P)\, \psi(X, Z, P)\, dZ.$$

For the elasticities relating to the observed distribution one gets

$$(2) \quad E_{\bar{F}X} \cdot \bar{F} = \frac{X \delta \bar{F}}{\delta X} = -X \int_0^\infty f(Z, P) \cdot \frac{\delta \psi(X, Z, P)}{\delta X}\, dZ = -\int_0^\infty f \frac{\delta \psi}{\delta \log X} \cdot dZ.$$

$$(3) \qquad E_{\bar{F}P} \cdot \bar{F} = -P \int_0^\infty \frac{\delta f}{\delta P} \psi\, dZ - P \int_0^\infty f \frac{\delta \psi}{\delta P}\, dZ.$$

By partial integration of the first integral in (3), one gets

$$(4) \quad E_{\bar{F}P} \cdot \bar{F} = P \int_0^\infty \frac{\delta F}{\delta P} \frac{\delta \psi}{\delta Z}\, dZ - P \int_0^\infty f \frac{\delta \psi}{\delta P}\, dZ; \text{ putting } \begin{cases} \psi(X, \infty, P) = 0 \\ \dfrac{\delta F(0, P)}{\delta P} = 0 \end{cases}.$$

Let the elasticity for the average amount purchased by a person with an average consumption of Z at price P be $E(Z, P)$. One then has, according to II, § 3, (2)

$$P \frac{\delta F}{\delta P} = - E(Z,P) \cdot \frac{\delta F}{\delta Z} \cdot Z = E(Z,P) \cdot f(Z,P) Z .$$

Introducing this expression in (4) and denoting the expression for the observed distribution corresponding to elasticity E by $\overline{E}$, one gets

$$(5)\quad E(X,P) = - \frac{E_{FP}}{E_{FX}} = \frac{\int_0^\infty E(Z,P) f(Z,P) \frac{\delta \psi(X,Z,P)}{\delta \log Z} dZ \quad \int_0^\infty f \frac{\delta \psi}{\delta \log P} \cdot dZ}{\int_0^\infty f \frac{\delta \psi}{\delta \log X} \cdot dZ \qquad \int_0^\infty f \frac{\delta \psi}{\delta \log X} \cdot dZ} .$$

We shall now assume that the distribution function ψ is a logarithmico-normal distribution and that this distribution is independent of the price. Let us further suppose that the dispersion, σ, of the logarithms of the quantities purchased is independent of the mean value of these logarithms.

According to II, § 2, (3) one thus has

$$\psi(X,Z,P) = \frac{1}{\sigma\sqrt{2\pi}} \int_0^X \frac{1}{X} \exp - \frac{1}{2} \left(\frac{\log X - \log Z + \frac{\sigma^2}{2}}{\sigma} \right) dX ,$$

or

$$\psi(X,Z,P) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\log X - \log Z + \frac{\sigma^2}{2}} \exp - \frac{1}{2} \frac{X^2}{\sigma^2} dX .$$

Introducing

$$\varphi(X) = \frac{1}{\sqrt{2\pi}} \exp - \frac{1}{2} X^2$$

one gets

$$\frac{\delta \psi}{\delta \log X} = - \frac{\delta \psi}{\delta \log Z} = \varphi \left(\frac{\log X - \log Z + \frac{\sigma^2}{2}}{\sigma} \right) .$$

Introducing this relation in (5) one has

$$\overline{E(X,P)} = \frac{\int_0^\infty E(Z,P) f\varphi\, dZ}{\int_0^\infty f\varphi\, dZ} \qquad (6)$$

If the dispersion σ in the normal logarithmic distribution is small, it follows from the weighting system in the average value (6) that the elasticity $\bar{E}$ will agree approximately with the elasticity E.

For elasticity in the case of consumption of a rationed commodity, with A denoting the size of the ration, one has

$$\overline{E^r} = \frac{\delta \bar{K}}{\delta P} \frac{1}{K} = \frac{1}{K} \int_0^A P \frac{\delta \bar{F}}{\delta P}\, dZ - \frac{\int_0^A M(E) Z \frac{\delta \bar{F}}{\delta A}\, dZ}{\int_0^A Z \frac{\delta \bar{F}}{\delta A}\, dZ} \cdot \frac{\bar{K} - AF}{\bar{K}},$$

with

$$M(E) = \frac{\int_0^\infty E(Z,P) f\varphi\, dZ}{\int_0^\infty f\varphi\, dZ}.$$

If the individual elasticities are equal to E for each consumer, then

$$M(E) = E = \bar{E} \text{ and } \overline{E^r} = E \cdot \frac{\bar{K} - A\bar{F}}{\bar{K}}.$$

This last relationship between the elasticity of a rationed commodity and the individual elasticity is fully analogous with the corresponding relationship in the case where the purchases are not regarded as stochastic variables.

Finally, it can be pointed out that if the distribution $F(X,P)$ is a logarithmico-normal distribution, and the same is true for ψ, then the observed distribution $\bar{F}$ is of the same type. The conditions are then fully analogous with the conditions discussed in § 2.

§ 5. Income elasticities. The relationship between individual elasticities and market elasticities

Even if the income series chosen for calculating the market income elasticity adequately expresses the fluctuations in the total purchasing power available for purchasing the commodity in question, it does not give any idea of the variation in the size of the fluctuations in income for different income categories.

A. We shall begin by illustrating the relationship between the individual income elasticities and the elasticity for the consumption of an unrationed commodity with regard to the total income (market elasticity).

Take n persons with incomes

$$i_1, i_2, \ldots, i_n; \ \Sigma i_\nu = I.$$

Let the corresponding purchases be

$$k_1, k_2, \ldots, k_n; \ \Sigma k_\nu = K.$$

The individual income elasticities $E_{k_\nu i_\nu}$ may be denoted by E_ν.

Further, let e_ν denote the elasticity $E_{i_\nu I}$, that is to say the elasticity of income i_ν with respect to the market income I.

For the market elasticity one then gets[1]

$$(1) \qquad \frac{I\,dK}{K\,dI} = E_{KI} = \frac{1}{\Sigma k_\nu} \Sigma E_\nu k_\nu \cdot \frac{di_\nu}{i_\nu} \frac{I}{dI} = \frac{1}{\Sigma k_\nu} \Sigma E_\nu k_\nu e_\nu .$$

If the incomes vary in the same proportion, that is to say

$$\frac{di_\nu}{i_\nu} = \frac{dI}{I}; \quad (\nu = 1, 2, \ldots, n),$$

one gets

$$E_{KI} = \frac{1}{K} \Sigma E_\nu k_\nu .$$

[1] H. Wold, »A Synthesis of Pure Demand Analysis», Part III, § 56. Skandinavisk Aktuarietidskrift 1944.

In such case, the income elasticity for the total consumption is an average value of the individual income elasticities, weighted by the quantities consumed.

If the individual elasticities are equally great and equal to E_{ki}, one gets

$$E_{KI} = E_{ki} \frac{1}{\Sigma k_\nu} \Sigma k_\nu e_\nu .$$

Suppose now that a functional relationship exists between consumption and income, that is to say,

$$k_\nu = A\, i_\nu^{E_{ki}} .$$

One then gets

$$E_{KI} = E_{ki} \frac{(\Sigma i_\nu)\left(\Sigma i_\nu^{E_{ki}} \frac{di_\nu}{i_\nu}\right)}{\left(\Sigma i_\nu \frac{di_\nu}{i_\nu}\right)(\Sigma i_\nu^{E_{ki}})} . \tag{2}$$

If the individual income elasticities $= 1$, one obtains from this

$$E_{KI} = E_{ki} = 1 . \tag{3}$$

Further, if the elasticities e_ν are uncorrelated with the consumption k_ν and the income i_ν, one gets

$$\frac{1}{\Sigma k_\nu} \Sigma e_\nu k_\nu = \frac{\Sigma e_\nu}{n} = \frac{e_\nu i_\nu}{\Sigma i_\nu} = 1 ,$$

that is to say

$$E_{KI} = E_{ki} .$$

In these cases the elasticity of the total consumption agrees with the individual elasticity.

On the other hand, if the change in income depends on the level of income in such a way that if

$$i_\nu \gtreqless i_\mu \text{ then } \frac{di_\nu}{i_\nu} \gtreqless \frac{di_\mu}{i_\mu},$$

one then gets[1]

$$\frac{\Sigma i_\nu^{E_{ki}} \frac{di_\nu}{i_\nu}}{\Sigma i_\nu^{E_{ki}}} < \frac{\Sigma i_\nu \frac{di_\nu}{i_\nu}}{\Sigma i_\nu} \text{ if } E_{ki} < 1. \tag{4}$$

If E_{ki} is greater than 1, the reverse sign of inequality holds. From (2) one then obtains

$$E_{KI} < E_{ki} \text{ if } E_{ki} < 1;$$
$$E_{KI} > E_{ki} \text{ if } E_{ki} > 1.$$

To express the same thing in words: If the change in income, expressed as a percentage, does not decrease with increased income, then the elasticity for total consumption with regard to total income will be less than the individual income elasticity if this is less than one. If the individual elasticity is greater than one, then the market elasticity will be greater than the individual elasticity.

If the change in income, expressed as a percentage, does not increase with increased income, the reverse relationship holds. Thus, in this case of equalization of income, the market elasticity will lie nearer 1 than the individual elasticity.

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[1] The inequality (4) is obtained from Tchebychef's inequality (see for instance Hardy-Littlewood-Pólya, "Inequalities", Cambridge 1935, § 2. 17) which says that

$$\left(\frac{\Sigma P a^r}{\Sigma P}\right)^{\frac{1}{r}} \left(\frac{\Sigma P b^r}{\Sigma P}\right)^{\frac{1}{r}} \lessgtr \left(\frac{\Sigma P a^r b^r}{\Sigma P}\right)^{\frac{1}{r}} \text{ if } (a_\mu - a_\nu)(b_\mu - b_\nu) \gtrless 0.$$

If one takes $r = 1$, $P = i_\nu^{E_{ki}}$, $a = \frac{di_\nu}{i_\nu}$, $b = i_\nu^{1-E_{ki}}$ one arrives at (4).

When the individual elasticities are unequal, the relationship is more complicated. In such case, we use formula (1) as a basis for further calculations. We obtain the following cases:

For $i_\nu \gtreqless i_\mu$; $E_\nu k_\nu \gtreqless E_\mu k_\mu$; $e_\nu \gtreqless e_\mu$, let the market elast. be E'_{KI}.
„ $i_\nu \gtreqless i_\mu$; $E_\nu k_\nu \gtreqless E_\mu k_\mu$; $e_\nu \lesseqgtr e_\mu$, „ „ „ „ „ E''_{KI}.
„ $i_\nu \gtreqless i_\mu$; $E_\nu k_\nu < E_\mu k_\mu$; $e_\nu \gtreqless e_\mu$, „ „ „ „ „ E'''_{KI}.
„ $i_\nu \gtreqless i_\mu$; $E_\nu k_\nu < E_\mu k_\mu$; $e_\nu \gtreqless e_\mu$, „ „ „ „ „ E''''_{KI}.

One then has $E'_{KI} \gtreqless E''_{KI}$; $E''''_{KI} > E'''_{KI}$,
where the equality sign is valid if $E_\nu k_\nu = E_\mu k_\mu$.

The first of these inequalities is naturally valid even in the case where individual elasticity increases with income and where the income elasticities are assumed to be positive. Thus, if the elasticities increase with income, then the market elasticity obtained, let us say, as the result of an increase in income that on a percentage basis is greatest for the low incomes, will be less than the market elasticity that is obtained when the increase, on a percentage basis, is greatest for the high incomes. If the elasticities decrease with increasing income, the effect of changes in the distribution of the increases in income will be dependent on how sharply the individual elasticities decrease compared with the corresponding quantities consumed.

B. If the commodity in question is rationed in the manner previously described, the relationship between the individual elasticities and the market elasticity will, generally speaking, be even more complicated.

1. In a special case where the group of consumers under consideration consists of persons with incomes of the same size and who have the same changes in income, the income elasticities can be interpreted in the same way as the price elasticities.

2. Suppose that of n consumers, μ of them take out their whole ration and let

$$k_{n-\mu+1} = k_{n-\mu+2} = \ldots = k_n = 1; \quad \sum_{\nu=1}^{n-\mu} i_\nu = I_1.$$

Let E^r_{KI} represent the market elasticity of the rationed commodity. One then has

$$E^r_{KI} = \frac{1}{\sum_1^n k_\nu} \cdot \frac{I}{dI} \sum_{\nu=1}^{n-\mu} E_\nu k_\nu \frac{di_\nu}{i_\nu} = \left[\frac{1}{\sum_1^{n-\mu} k_\nu} \sum_1^{n-\mu} E_\nu k_\nu \frac{di_\nu}{i_\nu} \frac{I_1}{dI_1} \right] \frac{\sum_{\nu=1}^{n-\mu} k_\nu}{\sum_1^n k_\nu} \frac{dI_1}{I_1} \frac{I}{dI}.$$

The expression in brackets has been written in the form of a market elasticity according to (1).

Incidentally, it can be mentioned that at the points where the number of consumers who do not take out their full ration change, the market elasticity does not exist.

The market elasticity of a commodity is thus equal to the product of the market elasticity for those consumers who do not take out the whole of their ration, the relationship between the purchases of these consumers and the total consumption, and also the elasticity for the income of these consumers with regard to the total income.

3. Finally, we shall study the case where the size of the ration depends on the income. Calculating the income elasticity one has

$$\frac{dK}{dI}\frac{I}{K} = \frac{I}{K}\left[\frac{\delta K}{\delta I} + \frac{\delta K}{\delta A}\frac{dA}{dI}\right] = E^r_{KI} + E_{KA}\frac{dA}{dI}\frac{I}{A}.$$

If the elasticity for the purchasing frequency is now introduced, that is to say

$$\frac{dUI}{dIU} = \frac{dKI}{dIK} - \frac{dAI}{dIA},$$

one gets

$$\frac{dKI}{dIK} = \frac{E^r_{KI}}{1-E_{KA}} - \frac{E_{KA}}{1-E_{KA}}\frac{dUI}{dIU};$$

$$\frac{dAI}{dIA} = \frac{E^r_{KI}}{1-E_{KA}} - \frac{1}{1-E_{KA}}\frac{dUI}{dIU}.$$

Thus, if the individual incomes and the changes in income are of the same magnitude, one gets the following in regard to the meaning of ration elasticity E_{KA}. If the purchasing frequencies are constant in respect to the income, the income

elasticity will agree with the market elasticity for those consumers who take out their full rations. In this case the market elasticity also agrees with the elasticity of the ration with respect to income. If the elasticity for the purchasing frequency is greater or less than zero respectively, then the income elasticity will be respectively less or greater than the market elasticity.

§ 6. A method for calculating the effect of altering the size of rations

A. Let the ration elasticity be regarded as a function of the purchasing frequency, that is to say

$$E_{KA} = \frac{F}{U} = \frac{G(U)}{U}.$$

The question is now whether this is an one-valued function of price, income and size of ration. If it is, a change in the purchasing frequency will give the same value for F, irrespective of whether the change in purchasing frequency is the result of a change in price, income or size of rations. In such case, the following conditions must be fulfilled.

$$(1) \qquad \frac{\delta F}{\delta P} = \frac{\delta F}{\delta I} = \frac{\delta F}{\delta A}.$$
$$\frac{\delta U}{\delta P} \quad \frac{\delta U}{\delta I} \quad \frac{\delta U}{\delta A}$$

Let $E_{KP}(X)$ and $E_{KI}(X)$ be the individual price and income elasticities, respectively.

Using the same designations as before, one gets, according to II, § 3, (2).

$$\frac{\delta F}{\delta P} = -E_{KP}(A)\frac{F}{P}E_{FA}; \quad \frac{\delta U}{\delta P} = \frac{K}{AP}E'_{KP},$$

that is to say

$$\frac{\delta F}{\delta P} = -\frac{E_{KP}(A)}{E^r_{KP}}\frac{AF}{K}E_{FA}.$$
$$\frac{\delta U}{\delta P}$$

In the same way

$$\frac{\frac{\delta F}{\delta I}}{\frac{\delta U}{\delta I}} = -\frac{E_{KI}(A)}{E^r_{KI}}\,\frac{AF}{K}E_{FA}; \qquad \frac{\frac{\delta F}{\delta A}}{\frac{\delta U}{\delta A}} = -\frac{AF}{K-AF}E_{FA}.$$

Then the conditions in (1) are

$$\frac{E_{KP}(A)}{E^r_{KP}} = \frac{E_{KI}(A)}{E^r_{KI}} = \frac{K}{K-AF}.$$

According to previous results, these conditions are fulfilled when the individual price elasticities are of the same magnitude; the same is true of the individual income elasticities and when the incomes of the consumers vary in the same proportion.

As an example we may use the results of an investigation of liquor purchases at the Malmköping Liquor Company referred to in § 7.

To obtain an expression of the function $G(U)$ let

$$F = G(U) = a_0 + a_1 U + a_2 U^2 + a_3 U^3 + a_4 U^4.$$

When $U = 0$, then $F = 0$, or $a_0 = 0$.

Thus

$$\frac{G(U)}{U} = E_{KA} = a_1 + a_2 U + a_3 U^2 + a_4 U^3.$$

The coefficients are determined by allowing the function to pass through the points below, which have been chosen on the basis of the values in § 7.

$$\begin{cases} U = 0 \\ E_{KA} = 0 \end{cases}; \quad \begin{cases} U = 0.6 \\ E_{KA} = 0.2 \end{cases}; \quad \begin{cases} U = 0.9 \\ E_{KA} = 0.7 \end{cases}; \quad \begin{cases} U = 1 \\ E_{KA} = 1 \end{cases}.$$

One gets

$$E_{KA} = \frac{F}{U} = U(0.43 - 1.26\,U + 1.83\,U^2).$$

For certain values of the purchasing frequency one gets the following values of the quotient of the price elasticity of the demand for the rationed commodity and the individual elasticity, according to II, § 1, (5).

U	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.50
$\frac{E'}{E}$	0.30	0.43	0.53	0.61	0.69	0.75	0.80	0.87

The second row can also be interpreted as the elasticity for the rationed commodity when the individual elasticity is 1.

B. If one assumes that the ration elasticity is a one-valued function of the purchasing frequency, the problem of calculating the effect of a change in the rations becomes a matter of solving the differential equation

$$\frac{dK}{dA} = F = G(U) = G\left(\frac{K}{A}\right).$$

A numerical integration of the above equation can be performed as follows:

Suppose that the size of the ration is A_0 and the average consumption K_0. Now let the size of the ration change from A_0 to A and the consumption from K_0 to the unknown quantity K.

Let us imagine that the difference between the rations, $A_0 - A$, is split up into n parts

$$|A_0;\ A_1|;\ |A_1;\ A_2| \ldots |A_{n-1};\ A|.$$

Let the consumed quantity K be expressed by

$$K = K_0 + (A_1 - A_0)\, G[U(A_{\Theta_1})] + (A_2 - A_1)\, G[U(A_{\Theta_2})] + \ldots$$
$$\ldots + (A - A_{n-1})\, G[U(A_{\Theta_n})],$$

where A_{Θ_ν} lies between $A_{\nu-1}$ and A_ν, and $U(A_{\Theta_\nu})$ is the frequency of purchasing when the size of the ration is A_{Θ_ν}.

When $A < A''$ is $U(A') > U(A'')$.

Then

$$K_0 + (A_1 - A_0)\, G[U(A_1)] + \ldots + (A - A_{n-1})\, G[U(A)] = K' \leqq K \leqq K_0 + (A_1 - A_0)\, G[U(A_0)] + \ldots + A - A_{n-1})\, G[U(A_{n-1})] = K''.$$

According to the investigation of liquor purchases at Malmköping, mentioned above, the average amount of liquor bought in 1934 by ration book holders entitled to 4 litres a month was 31.12 litres. As a numerical illustration we shall now work out an estimate of the average amount these ration book holders would buy if their rations were decreased to 3 litres per month.

Let the difference in the annual rations, 48-36 litres, be split up into four equal parts, each of three litres. The calculations can be followed from the following table:

A	K'	U'	F'	K''	U''	F''
Ration in litres per year	Consumption. Upper limit	Purchasing frequency. Upper limit	Upper limit	Consumption. Lower limit	Purchasing frequency. Lower limit	Lower limit
48	31.12	$\frac{31.12}{48} = 0.648$	0.160	31.12	0.648	0.160
45	$31.12 - 3 \cdot 0.160 = 30.64$	$\frac{30.64}{45} = 0.681$	0.195	$31.12 - 3 \cdot 0.195 = 30.54$	$\frac{30.54}{45} = 0.679$	0.193
42	$30.64 - 3 \cdot 0.193 = 30.06$	0.716	0.239	29.82	0.710	0.231
39	29.37	0.753	0.294	28.94	0.742	0.277
36	28.54	0.793	0.366	27.84	0.773	

Thus, according to the above calculations, a reduction in rations from 4 to 3 litres a month would give an average annual consumption of between 27.84 and 28.54 litres per ration book holder, and a purchasing frequency of between 0.773 and 0.793.

§ 7. Elasticities in the case of differentiated rationing

When the rationing of a commodity is carried out in such a way that the consumers are divided into different groups with different rations, we say that the rationing is differentiated.

In this paragraph we shall consider the relationship between an elasticity for total demand and the corresponding elasticities for the different ration groups.

Suppose that one has N ration groups. Let us introduce the following terms.

A_ν = size of ration for ration group ν,
n_ν = relative number of consumers in ν,
k_ν = average consumption in ν,
f_ν = relative number of consumers in ν, who take out their full rations,
K = average market consumption,
A = average size of ration for all groups.

For the elasticity of the market consumption with respect to price one gets

$$E_{KP}^{r} = \frac{P\,dK}{K\,dP} = \frac{1}{K}\Sigma n_\nu k_\nu E_{k_\nu P}^{r}. \tag{1}$$

Comparing now with (5) in II, § 3, it is seen from (1), that even if the rationing is differentiated we get the same relationship between the market elasticity for the demand of the rationed commodity and the individual elasticities as expressed in II, § 3, (5).

Denoting the weighted mean of the individual elasticities of those consumers, who do not take out their full rations by $M[E]$ we have

$$E_{KP}^{r} = M[E]\frac{\Sigma n_\nu k_\nu - \Sigma A_\nu n_\nu f_\nu}{\Sigma n_\nu k_\nu} = M[E]\left[1 - \frac{\Sigma A_\nu n_\nu f_\nu}{\Sigma n_\nu k_\nu}\right]. \tag{2}$$

On the other hand, we have

$$F = \Sigma n_\nu f_\nu; \quad U = \frac{\Sigma n_\nu k_\nu}{\Sigma n_\nu A_\nu}, \tag{3}$$

and thus

$$E_{KP}^{r} = M[E]\left[1 - \frac{F}{U}\frac{\Sigma A_\nu n_\nu f_\nu}{\Sigma n_\nu f_\nu \Sigma n_\nu A_\nu}\right]. \tag{4}$$

If the dispersion in the relative numbers f is small, we have

$$E^r_{KP} \sim M[E]\left[1-\frac{E}{U}\right].$$

For example, we may quote the following results of an investigation of liquor purchases at the Malmköping liquor company during 1930-1934.[1] The figures give the number of ration book holders (f_ν) in percentages within the different ration groups who have taken out their rations in full, and also the ratio between the average quantity purchased and the ration, that is to say the purchasing frequency (U) expressed as a percentage.

Ration (litres pr month)	4			3			2			1		
Year	Nos of r.b.h.	f_ν	U_ν	Nos of r.b.h.	f_ν	U_ν	Nos of r.b.h.	f_ν	U_ν	Nos of r.b.h.	f_ν	U_ν
1930	1069	57.6	88.1	431	61.5	91.8	1185	61.9	89.7	362	54.7	85.2
1931	1031	60.5	88.5	424	64.9	93.2	1191	63.0	88.8	376	56.3	84.3
1932	1019	33.0	79.2	448	31.3	81.7	1247	32.6	78.4	453	34.7	74.4
1933	984	11.7	60.6	447	8.6	58.4	1232	7.2	53.4	486	10.9	59.7
1934	953	16.7	64.8	439	14.1	63.7	1205	11.3	58.7	486	11.9	54.7

For all ration book holders investigated, one gets

Year	F	U	$1-\frac{F}{U}$	$\frac{E^r_{KP}}{M[E]}$ according to (4)
1930	59.5	89.0	0.332	0.334
1931	61.6	88.9	0.308	0.307
1932	32.8	79.8	0.585	0.587
1933	9.4	57.5	0.837	0.830
1934	13.5	62.2	0.784	0.769

[1] K. G. Brolin, »Motboksinnehavarnas spritinköp vid Malmköpings nya spritbolag åren 1930 -34 i statistisk belysning» (A Statistical Survey of Liquor Purchased by Ration Book Holders at Malmköping Liquor Company during 1930—1934). Ms. at the Institute of Statistics, University of Upsala.

For the elasticity of the market consumption with respect to the average ration one gets

$$E_{KA} = \frac{d\,K}{d\,A}\,\frac{A}{K} = \frac{\Sigma n_\nu\, d\,k_\nu}{\Sigma n_\nu d\,A_\nu}\,\frac{\Sigma n_\nu A_\nu}{\Sigma n_\nu k_\nu}.$$

But

$$\frac{d\,k_\nu}{d\,A_\nu} = f_\nu.$$

Thus

$$E_{KA} = \frac{\Sigma n_\nu f_\nu\, d A_\nu}{\Sigma n_\nu\, d A_\nu}\,\frac{\Sigma n_\nu A_\nu}{\Sigma n_\nu\, k_\nu}.$$

The market ration elasticity obviously depends on how the rations are changed.

If the rations are changed in the same ratio, that is to say

$$\frac{d\,A_\nu}{A_\nu} = \frac{d\,A}{A}; \qquad (\nu = 1, 2, \ldots, N),$$

one gets, according to (2)

$$E_{KP}^{r} = M[E][1 - E_{KA}].$$

If the rations are changed by the same amount, that is to say

$$d\,A_\nu = d\,A; \quad (\nu = 1, 2, \ldots, N),$$

one gets, according to (3)

$$E_{KA} = 1 - \frac{F}{U}.$$

§ 8. Elasticity with regard to size of rations as given by a time series. The results of conditional regression analysis

Changes in the average ration of a commodity that is differentially rationed may arise from several causes. The rations within one or several rationing classes can be changed on certain occasions, resulting in sudden changes in the average ration. Changes in the size of the average ration can also be caused by the original group of consumers being increased by the addition of new consumers with for example, a lower demand (or less »need»for the commodity in question). Taking this into account,

the average ration for these consumers may be expected to be less than that of the other members of the consumer group, providing the rationing regulations are interpreted in the same way. The continual entry of customers of this kind into the consumer group would thus mean a continually decreasing average ration. On the other hand, providing the rationing regulations are always interpreted in the same way, the purchasing frequencis within the different rationing classes will remain unchanged. A decreasing average ration may also be caused by a more restrictive rationing policy being applied to the newcomers. These newcomers then are generally placed in lower rationing classes than corresponding consumers previously included in the consumer group. In this case, however, the purchasing frequency will increase. A decreasing average ration due to this cause is then comparable with a decreasing average ration due to successive decreases in the size of the rations. A decreasing average ration can, of course, be due also to the above causes acting in varying degrees.

Thus, in calculating the elasticity of consumption with regard to the size of the rations when the calculations are based on time series, it is also necessary to pay regard to the variations in the relative number of consumers within the different rationing classes.

Let T denote the average ration and t a time variable. Using the same terms as before, one then has

$$K = \sum_1^N n_\nu k_\nu; \quad T = \sum_1^N n_\nu A_\nu$$

and

$$E_{KT} = \frac{T}{K}\frac{dK}{dT} = \frac{\Sigma n_\nu A_\nu}{\Sigma n_\nu k_\nu}\left[\frac{\Sigma k_\nu \frac{dn_\nu}{dt}}{\Sigma A_\nu \frac{dn_\nu}{dt}} + \frac{\Sigma n_\nu \frac{dk_\nu}{dt}}{\frac{dT}{dt}}\right] =$$

$$= \frac{M_{W'}[U_\nu]}{M_{W''}[U_\nu]} + \frac{1}{M_{W''}[U_\nu]}\frac{\Sigma n_\nu A_\nu \frac{dU_\nu}{dt}}{\frac{dT}{dt}}; \quad \begin{cases} w_\nu' = A_\nu \frac{dn_\nu}{dt} \\ w_\nu'' = A_\nu n_\nu \end{cases}$$

where $M_W[U_\nu]$ denotes a weighted mean value for U_ν with the weights W_ν.

If the differentiated rationing is carried out in such a manner that the changes in the average ration depend only on the variations in the relative number of consumers in the rationing classes while the purchasing frequencies within these classes, on the other hand, remain unchanged, one will have

$$\frac{dU_\nu}{dt} = 0; \quad E_{KA} = \frac{M_{W'}[U_\nu]}{M_{W''}[U_\nu]}.$$

Hence, in such a case the elasticity with regard to the average ration can be represented as a ratio between two weighted average values of the purchasing frequencies within the different rationing classes. Moreover, if the rationing is carried out in such a way that these purchasing frequencies approximately agree, one has

$$E_{KT} \infty 1.$$

One can also express this by saying that as a result of the rationing policy, the stratification effect, which is discernible through changes in the average quantity purchased and the average ration, has been eliminated as far as the purchasing frequencies are concerned.

On the other hand, if the purchasing frequencies had been changed, this could be interpreted as a changes in the rationing regulations. The inequalities

$$\frac{dU_\nu}{dt} > 0; \quad (\nu = 1, 2, \ldots, N),$$

may thus be said to signify a tightening up of the rationing policy followed. The reverse inequality would indicate a relaxing of the retioning policy.

Supposing, as before, that the purchasing frequencies are equal, and also that

$$\frac{dU_\nu}{dt} = \frac{dU}{dt}; \quad (\nu = 1, 2, \ldots, N),$$

one has

$$E_{KT} = 1 + \frac{dU}{dt}\frac{dt}{dT}\frac{T}{U}.$$

For a decreasing or increasing average ration respectively, one has

$$\frac{dT}{dt} < 0 \text{ and } \frac{dT}{dt} > 0 \text{ respectively.}$$

The above relations show that if the average ration decreases, one gets an elasticity for the average quantity purchased with regard to the average ration that amounts to less than 1, in the case of a tightening up of rationing policy. With a more liberal interpretation of the rationing regulations, the elasticity will, under the same conditions, be greater than 1. If the average ration increases, the relationship will be reversed.

During the period 1923-40, the average liquor ration decreased; during the same time the number of ration book holders increased. Suppose the number of ration book holders increased with consumers who, under unchanged rationing policy as defined above, receive an average ration that is less than the general average. Then, if the reduction in the average ration can be explained solely on this ground, the elasticity for the average quantity purchased with regard to the average ration will be approximately equal to 1. This means that the purchasing frequency can be regarded as a dependent variable in regression analysis.

In so far as the decreasing average ration really implies a reduction in the rations in the sense that the consumers are generally placed in a lower rationing class than formerly, the rationing elasticity will be less than one.

Regard can be taken of these factors in regression analysis by giving the rationing elasticity a definite value in advance.[1] By fixing the rationing elasticity at 1 or 0.9, respectively, and using the same signs as in I, § 1 the coefficients in the following table are obtained.

It is clear from the table that the correlation between purchasing frequency and the different price series is less than between the quantity purchased and the prices (cf. I, § 4). When the income variable is introduced, the coefficient of correlation, R, is markedly increased. The greatest coefficient of correlation is obtained by choosing the average price series P''.

[1] Regarding conditional regression analysis, see Wold 1940, § 10.

Elasticities obtained by fixing the rationing elasticity at a given value.

Period	Relationship referring to I, § 4	Price	Income		log T	log P	log I	R
1923-39	I, 1	P'_N	—	$\log K_0 = a_0 +$	1.0	— 0.234	—	0.694
				$\log K_0 = a_0 +$	0.9	— 0.253	—	—
1923-39	I, 6	P'_N	I_N	$\log K_0 = a_0 +$	1.0	— 0.372	0.460	0.985
				$\log K_0 = a_0 +$	0.9	— 0.373	0.385	—
1923-39	II, 1	P'_R	—	$\log K_0 = a_0 +$	1.0	— 0.198	—	0.671
				$\log K_0 = a_0 +$	0.9	— 0.219	—	—
1923-39	II, 3	P'_R	I_R	$\log K_0 = a_0 +$	1.0	— 0.336	0.422	0.966
				$\log K_0 = a_0 +$	0.9	— 0.346	0.387	—
1923-38	III, 1	P''_R	—	$\log K_0 = a_0 +$	1.0	— 0.195		0.670
				$\log K_0 = a_0 +$	0.9	— 0.213		
1923-38	III, 2	P''_R	I_R	$\log K_0 = a_0 +$	1.0	— 0.301	0.454	0.984
				$\log K_0 = a_0 +$	0.9	— 0.310	0.414	—
1923-39	III, 4	P'''_R	I_R	$\log K_0 = a_0 +$	1.0	— 0.239	0.458	0.956
				$\log K_0 = a_0 +$	0.9	— 0.247	0.425	—
1923-40	IV, 1	P'_R	—	$\log K_0 = a_0 +$	1.0	— 0.279	—	0.801
				$\log K_0 = a_0 +$	0.9	— 0.297	—	—
1923-40	IV, 3	P'_R	I_R	$\log K_0 = a_0 +$	1.0	— 0.409	0.452	0.953
				$\log K_0 = a_0 +$	0.9	— 0.416	0.416	—
1923-40	IV, 4	P'''_R	I_R	$\log K_0 = a_0 +$	1.0	— 0.298	0.491	0.806
				$\log K_0 = a_0 +$	0.9	— 0.304	0.458	—

One sees too, that the use of purchasing frequency as an independent variable results in a slight decrease in the numerical values of the price elasticities (cf. I, § 4). Thus, these values are practically independent of the assumptions made concerning the interpretation of the rationing regulations.

CHAPTER III

From the results in the preceding chapter, it would seem reasonable to suppose that the price elasticity for liquor sold by bottle stores will vary with purchasing frequency. The lower the purchasing frequency, the higher the numerical value one can expect for the elasticity of the rationed commodity. This was also found to be the case in chapter I where regression analysis gave a numerically smaller price elasticity for the period 1923-39 than when figures for 1940 were also included.

Accordingly, we shall examine the variations of consumption with price in greater detail.

One can get an idea of the variation of elasticity by comparing price elasticities calculated for short periods with varying purchasing frequency. In such case it is simplest, having arrived at a result for the income elasticities, to work out the price elasticities from two values for quantity.

These two values for quantity should both relate to periods during which there has been no change in price. For, if the calculation of elasticity is based on figures for quantity and price that are averages for periods that have not been free from price changes, the results of the calculation may, to some extent, be dependent on other factors besides the elasticities of demand.

The systematic effect, which in this case will make its presence felt, is discussed in § 1, and can be described as follows:

Suppose one wants to estimate the constant price elasticity of the demand for a commodity from the average quantity and the average price for two periods, of which only one is

free from price changes. Providing the relationship between price and consumption is functional, the calculated value of price elasticity will, in the case of an under-elastic demand, be numerically less, in the case of an over-elastic demand numerically greater than the real elasticity. If the demand is normal-elastic, both elasticities will egree.

This systematic effect can, moreover, be amplified in such cases where the size of the consumed quantities is affected by, let us say, seasonal fluctuations or purchases made with the intention of »stocking up» before an anticipated rise in price.

The arguments put forward in § 1 hold true, irrespective of whether one is concerned with a demand for a rationed commodity or an unrationed commodity. Further, the independent variable need not be the price, it may of course be something else, such as the income.

Thus, in working out price elasticities, one should, if possible, use values relating to periods of stable price. However, in order that the quantity values obtained in this manner shall be comparable, it is necessary to eliminate the possibility of seasonal variations in them.

By means of the methods given in § 2, a calculation is made in § 3 of the seasonal fluctuations in purchasing frequency. These fluctuations, which are of interest in themselves, are particularly marked, with large purchases in June and December and small purchases in January and February.

The seasonal fluctuations in purchasing frequencies show greater dispersion for the period 1933-38 than for the period 1926-31. This means that the decrease in purchasing has been greatest for the months when little liquor has been purchased.

By the aid of these seasonal fluctuations, comparable monthly purchasing frequencies for certain periods when the prices of liquor have been stable have been worked out in § 4.

Another difficulty in determining price elasticity lies in the determination of the change in incomes, and the effect of these changes in the size of the purchasing frequency.

In passing, it can be said that these price elasticities are an expression of the changes in purchases during the months immediately following a change in price. Elasticities of this kind are sometimes called short run elasticities, as distinct from

long run elasticities, which express the effect of a price change on demand, after this has been stabilizied at the new price level.[1]

The results of the calculations suggest the expected numerical increase in price elasticity with decreasing purchasing frequency.

In § 5, a corresponding calculation has been made for wine sold by bottle stores.

Since liquor is not a unit commodity, but consists of a large number of different qualities at varying prices, changes in price and income produce, as mentioned previously, not only a change in the quantity purchased, but also a change in the distribution of purchases among the different kinds of liquor. Accordingly, in §§ 6 and 7, an estimate has been made of the seasonal fluctuations in the amount of two kinds of liquor purchased each month. On the basis of these seasonal fluctuations, it has been possible to work out comparable consumption figures for the »price stable» periods discussed previously. The variable quantity used in this case was the amount of liquor in question that was purchased, expressed as a percentage of total liquor saldes.

The changes in the development of consumption may now be described briefly as follows:

During the period 1925-30, the purchasing frequency increased simultaneously with increasing incomes, the price of liquor meanwhile remaining relatively stable. This increase in purchasing frequency ended in 1931, however, in conjunction with a weakening of purchasing power. On three occasions during the period 1932-33, prices were raised, namely on Feb. 1, 1932, by 60 öre, On July 1, 1932, by 50 öre, and on Feb. 1, 1933, by 60 öre. At the same time, purchasing power decreased. However, it would seem that the decline in purchasing during this period was concentrated to the months when prices were raised. The reaction was least after the first price increase, greatest after the last. The increase in purchasing which followed was replaced by a marked decrease in purchasing following a new price increase of 3 kr. on Nov. 1, 1939. It would

[1] Cf. Wold 1940, § 25.

seem that many ration book halders have taken the trouble to »stock up» immediately before pending price increases, for purchasing has been high during months immediately preceding price increases, low during months immediately following increases.

During the period 1925-30 there was a certain change-over from purchasing cheap to expensive kinds of liquor. The increase in prices during the period 1932-33 had the reverse effect. For the two kinds of liquor examined in §§ 6 and 7, this displacement in the distribution of consumption was most marked after the first two price increases. During the following years, the increase in purchasing power has principally taken the form of an increase in the quantity of liquor purchased.

§ 1. The use of average prices in calculating price elasticities

In the foregoing, figures for annual consumption have been set in relation to average prices obtained by dividing the total expenditure by the corresponding consumption, or by forming weighted average values with constant weights of similar average prices for the different main items of liquor. In the case of a change in price during a period (year), this method takes no account of the seasonal fluctuations in consumption, the effect on purchasing of the consumers' anticipation of price changes, or of reactions immediately following price changes. Moreover, in certain cases it would seem that the use of average prices in elasticity calculations can bring about a systematic effect on the results of the calculation. We shall now look at these conditions more closely.

A. Let us suppose that there is a functional relationship between the consumption per unit of time and the price of a commodity, that is

$$K = \varphi^*(P) \text{ or } P = \varphi(K).$$

Let us obtain an average price, P, over a period of $t_1 + t_2$ units of time. After t_1 units of time, the price is supposed to change

from $\varphi(K_1)$ to $\varphi(K_1+x)$, and the consumption from K_1 to K_1+x per unit of time. As the average price P is obtained by dividing the expenditure O by the corresponding consumption K during the time interval t_1+t_2, one gets

$$P = \frac{a_1 K_1 \varphi(K_1) + a_2 (K_1 + x)\varphi(K_1+x)}{K_1 + a_2 x}; \text{ where } \begin{cases} a_1 = \dfrac{t_1}{t_1+t_2}; \\ a_2 = \dfrac{t_2}{t_1+t_2}. \end{cases}$$

Now, let us regard the difference, S, between the functionally expressed price $\varphi(K_1+a_2x)$ and the average price $\bar{P}$.

$$S = \varphi(K_1 + a_2 x) - P = \frac{1}{K_1 + a_2 x} F(a_2),$$

where

$$F(a_2) = f(K_1 + a_2 x) - f(K_1) - a_2 [f(K_1+x) - f(K_1)],$$

and

$$f(x) = x f(x).$$

One finds that

$$F(a_2 = 0) = 0; \quad F(a_2 = 1) = 0,$$

and

$$F'_{a_2}(a_2 = 0) = -\frac{x^2}{2} f''(K_1 + \Theta_1 x); \quad 0 \leqq \Theta_1 \leqq 1;$$

$$F'_{a_2}(a_2 = 1) = +\frac{x^2}{2} f''(K_1 + \Theta_2 x); \quad 0 \leqq \Theta_2 \leqq 1;$$

$$F''_{a_2} = x^2 f''(K_1 + a_2 x).$$

Suppose, for instance, that the price elasticity during the price interval in question is constant and equal to E.

One then has

$$\varphi(x) = A x^{\frac{1}{E}} \text{ and } f(x) = A x^{\frac{1}{E}+1}; \quad A > 0; \quad E < 0.$$

Here

$$f'' > 0 \text{ if } E > -1;$$
$$f'' = 0 \text{ if } E = -1;$$
$$f'' < 0 \text{ if } E < -1,$$

and from the above relations one gets

$$S < 0 \text{ if } E > -1;$$
$$S = 0 \text{ if } E = -1;$$
$$S > 0 \text{ if } E < -1.$$

These inequalities mean that for constant price elasticity, the average price obtained by dividing the expenditure by the corresponding consumption is higher or lower respectively than the price that is functionally conditioned by consumption, if the demand is under- or over-elastic.

If the demand is normal-elastic, both prices will agree.

One can easily see that the corresponding relations also hold in the case where several price changes take place during the time interval with which one is dealing, providing, of course, that these price changes take place within that price interval in which the price elasticity has been assumed to be constant.

B. In the case of a price change anticipated by the consumers, the expenditure may decrease as a result of certain purchases being postponed until after the reduction in price, or being increased in the period immediately preceding a rise in price.

We shall first consider the conditions pertaining to a rise in price. Suppose that the consumers, anticipating a rise in prices, increase their purchases at the lower price P_1, by the quantity k_1. Suppose also that they decrease their purchases at the higher price P_2 by k_2.

Denoting the average price by $\overline{\overline{P}}$, one gets

$$\overline{\overline{P}} = \frac{K\overline{P} + k_1 P_1 - k_2 P_2}{K + k_1 - k_2}; \text{ where } \begin{cases} P_1 < \overline{P} < P_2; \\ P_1 < \overline{\overline{P}} < P_2. \end{cases}$$

Then

$$K(\bar{\bar{P}} - \bar{P}) = k_1(P_1 - \bar{P}) + k_2(\bar{\bar{P}} - P_2) < 0,$$

or

$$\bar{\bar{P}} < \bar{P}.$$

When certain purchases are postponed until after an anticipated reduction in price, one gets the same result.

If a price change produces no change in purchasing, or decreased purchasing, as a result of the causes considered here, one will have, in the case of an over-elastic demand

$$\bar{\bar{P}} < \bar{P} < \varphi(K) \leqq \varphi(K - k_1 + k_2).$$

Thus, in the case of over-elastic demand, a shift in purchasing of this kind produces an ampflication of the effect mentioned previously, which is encountered when calculating average prices. The condition is reversed in the case of an under-elastic demand.

A similar line of thought can be advanced in respect of the case where there are seasonal fluctuations in purchases during the period under consideration.

As an illustration, we may take the following example.

Let us suppose we have a commodity with a price elasticity of —2 for the price interval in question. The consumption of this commodity is studied during two periods of time. At the middle of the second period, the price is increased by 20 %. On the basis of average prices we are required to work out the price elasticity. In the case of the first period, let the consumption be $K_1 = AP_1^{-2}$ and the price P_1. For the second period, the consumption K_2 is given by

$$K_2 = \frac{1}{2} A P_1^{-2} + \frac{1}{2} A P_1^{-2} (1.2)^{-2} = 0.8472\, K_1.$$

For the average price P_2 one gets

$$P_2 = \frac{1}{K_2} \left[\frac{1}{2} P_1 K_1 + \frac{1}{2} \cdot 1.2\, P_1 K_1 (1.2)^{-2} \right] = 1.0820\, P_1;$$

$$E = \frac{\log K_1 - \log K_2}{\log P_1 - \log P_2} = -2.11.$$

Further, let us suppose that as a result of anticipated increases in prices the consumers buy in an additional stock which amounts to 0.1 K_2, and which produces a corresponding decrease in purchasing during the second half of the second period.

One has

$$\overline{P}_2 = \overline{P}_2 + 0.1\,(P_1 - 1.2\,P_1) = 1.0620\,P_1,$$

and

$$E' \quad 2.76.$$

Finally, if we suppose that as a result of seasonal fluctuations purchases during the first half of the second period amount to 1,05 K_2, and during the second half of the same period to 0.95 K_2, one will then have

$$E' \quad 3.27.$$

§ 2. Methods used for calculating seasonal fluctuations

The seasonal variations in a time series consist of more or less marked periodic fluctuations with, in most cases, a period length of one year. These fluctuations can be caused by periodic phenomena, such as meteorological factors, holidays etc. which affect consumption habits, production factors etc.

There are marked fluctuations in the amount of liquor bought during the different months of the year. The largest quantities are sold in June and especially in December, the smallest quantities in January and February.

The methods used for determining seasonal fluctuations vary in character, primarily with regard to the hypotheses on which they are based, but also in respect to the extent of the numerical calculations they lead to.[1]

For calculating the seasonal fluctuations we shall here use two methods.

[1] For a summary of the different methods, see O. Donner, »Die Saisonschwankungen als Problem der Konjunkturforschung», Vierteljahrshefte zur Konjunkturforschung, Sonderheft 6, Berlin, 1928.

H. Mendershausen, »Annual Survey of Statistical Technique: Methods of Computing and Eliminating Changing Seasonal Fluctuations», Econometrica. vol. 5, 1937, p. 234.

A. Hald, »The Decomposition of a Series of Observation», Copenhagen, 1948.

I. Let the time series under consideration, say u_{ik}, be given for every month during n years. Suppose that the time series is the sum of three components: trend, say T_{ik}, seasonal fluctuations, S_{ik}, and a remaining component, ε_{ik}. Thus

$$(1) \qquad u_{ik} = T_{ik} + S_{ik} + \varepsilon_{ik}; \quad \left(\begin{array}{l} i = 1, 2, \ldots, n \\ k = 1, 2, \ldots, 12 \end{array} \right).$$

Further, we shall assume that the seasonal fluctuations remain unchanged from year to year during the period under consideration, that is to say,

$$(2) \qquad S_{ik} = S_{(i+1)\,k} = S_k \text{ and further } \sum_{1}^{12} S_k = 0.$$

In arriving at a value for T_{ik}, a moving average covering 12 months is used, say m_{ik}.
Thus

$$m_{ik} = \frac{1}{12}\left[\frac{1}{2}u_{i(k-6)} + \sum_{r=-5}^{+5} u_{i(k+r)} + \frac{1}{2}u_{i(k+6)}\right],$$

where

$$u_{i(k\pm 12)} = u_{(i\pm 1)\,k}.$$

If the trend takes the form

$$T_{ik} = a_0 + a_1(12\,i + k) + a_3(12\,i + k)^3 + \ldots,$$

one gets

$$m_{ik} = T_{ik} + \frac{1}{12}\left[\frac{1}{2}\varepsilon_{i(k-6)} + \sum_{r=-5}^{+5} \varepsilon_{i(k+r)} + \frac{1}{2}\varepsilon_{i(k+6)}\right] = T_{ik} + \varepsilon,$$

and one has

$$m_{ik} \infty T_{ik}.$$

If, however, the trend takes the form

$$T_{ik} = a_0 + a_1(12\,i + k) + a_2(12\,i + k)^2,$$

one gets

$$m_{ik} = T_{ik} + \frac{73}{6}a_2 + \varepsilon.$$

The value for the moving average m_{ik} will thus over- or underestimate, respectively, the trend value T_{ik} depending on whether a_2 is positive or negative. Neglecting this effect, one can put

$$m_{ik} \approx T_{ik},$$

and

$$u_{ik} - m_{ik} = S_{ik} \approx S_k + \varepsilon_{ik}. \tag{3}$$

In this way one gets $(n\text{-}1)$ determinations of the seasonal component S_k. These determinations form a table with $12\,(n-1)$ values. An examination of this table will give an idea of the correctness of the supposition (2).

After this, one forms an average value for the determinations of S_k for each month. After correcting the average values in such a way that their sum total is zero, these values are then taken as an estimate of the seasonal component S_k.

The average value which is used in the following is an arithmetical average of the differences in (3), the greatest and smallest differences being omitted.

Summing up all the differences in the table, one gets

$$\sum_{i,k} \bar{S}_{ik} = \sum_{i,k} (u_{ik} - m_{ik}) =$$

$$= -\frac{1}{24}\left[\sum_{k=1}^{6} (2k-1)\,u_{1k} - \sum_{k=7}^{12} (25-2k)\,u_{1k} - \right.$$

$$\left. - \sum_{k=1}^{6} (2k-1)\,u_{nk} + \sum_{k=7}^{12} (25-2k)\,u_{nk}\right].$$

This expression, which can be used for checking the computations, is generally not equal to zero.

For instance, if one has

$$\begin{aligned} T_{1k} &= a_0 + a_1 k; \quad \varepsilon_{1k} = 0; \\ T_{nk} &= b_0 + b_1 k; \quad \varepsilon_{nk} = 0; \end{aligned} \qquad (k = 1, 2, \ldots, 12),$$

one gets

$$\sum_{i,k} (u_{ik} - m_{ik}) = \frac{73}{12}\,(b_1 - a_1).$$

Let us now look at the case where one or several of the values for μ_{ik} show a marked divergence from their normal value. For instance, if μ_{ik} is a series of purchased quantities of a commodity, it is possible that a sales strike may result in an abnormally small quantity of the commodity being sold. This results, firstly, in the corresponding difference S_{ik} being abnormally small. However, the abnormal figure for the amount purchased will, via the moving averages, affect even the six preceding and the six following differences, provided the small quantities purchased during the period of strike are not compensated by a corresponding increase in purchasing during the next period. The differences mentioned will be too large, as a result of the decreased averages. Consequently, it may be desirable in forming moving averages to neglect the decrease in purchases.

For the sake of simplicity, we can denote the period in question by 0, and the change in quantity by φ_0. Thus, one has

$$\bar{u}_0 = u_0 + \varphi_0$$

Let m_ν represent the moving averages calculated from the normal value u_0 and let us denote the moving averages actually observed by $\bar{m}_\nu$.

Thus

$$\bar{m}_\nu = m_\nu + \frac{\varphi_0}{12}; \quad (\nu = -5, -4, \ldots, 0, \ldots, +4, +5);$$

$$\bar{m}_\nu = m_\nu + \frac{\varphi_0}{24}; \quad (\nu = -6, +6).$$

We now form a difference D_0 in the following manner

$$D_0 = \bar{u}_0 - S_0 - \bar{m}_0 = u_0 + \varphi - S_0 - \left(m_0 + \frac{\varphi_0}{12}\right).$$

As

$$m_0 \infty u_0 - S_0,$$

one gets

$$D_0 \infty \frac{11}{12}\varphi_0.$$

Thus, one obtains the following corrections:

$$m_\nu \sim \bar{m}_\nu - \frac{D_0}{11}; \quad (\nu = -5, -4 \ldots +4, +5);$$

$$m_\nu \sim m_\nu - \frac{D_0}{22}; \quad (\nu = -6, +6).$$

The seasonal correction for the period in question, S_0, can be worked out from the table for the differences between the observed values and their moving averages, omitting the difference $\bar{u}_0 - \bar{m}_0$.

If one has

$$\bar{u}_0 = u_0 + \varphi_0; \quad \bar{u}_1 = u_1 + \varphi_1,$$

for the two consecutive periods, then one gets the following corrections by analogy:

$$m_{-6} \sim \bar{m}_{-6} - \left(\frac{11}{240} D_0 + \frac{1}{240} D_1\right);$$

$$m_{-5} \sim \bar{m}_{-5} - \left(\frac{23}{240} D_0 + \frac{13}{240} D_1\right);$$

$$m_\nu \sim \bar{m}_\nu - \left(\frac{D_0}{10} + \frac{D_1}{10}\right); \quad (\nu = -4, -3, \ldots, +3, +4);$$

$$m_{+6} \sim \bar{m}_6 - \left(\frac{13}{240} D_0 + \frac{23}{240} D_1\right);$$

$$m_{+7} \sim \bar{m}_7 - \left(\frac{1}{240} D_0 + \frac{11}{240} D_1\right),$$

where

$$D_0 = \bar{u}_0 - S_0 - \bar{m}_0;$$

$$D_1 = \bar{u}_1 - S_1 - \bar{m}_1.$$

II. The method of calculating the seasonal components on the basis of moving averages has the disadvantage that only the six first and six last elements of the time series are included in the moving averages giving no corresponding difference $\overline{S}_{ik}$.

Also, as has been pointed out earlier, the trend in the time series should have only a very small curvature. For a shorter time series with a greater curvature one can use the following method which also assumes that the seasonal fluctuations remain unchanged from year to year during the period under consideration.

Let us suppose that the given series can be written

$$u_{ik} = a_0 + a_1 X_{ik} + a_2 X_{ik}^2 + S_k + \varepsilon_{ik}$$

with

$$\sum_{1}^{12} S_k = 0.$$

If one takes

$$X_{ik} = 24\left(i - \frac{n}{2}\right) + 2k - 25$$

one gets

$$\sum_{i,k} X_{ik} = 0$$

and

$$\sum_{i,k} X_{ik}^3 = 0; \quad \sum_{k}\left(\sum_{i} X_{ik} \sum_{i} X_{ik}^2\right) = 0.$$

In seeking the values of the constants a_0, a_1, a_2 and S_k that minimize the sum of the squares of the deviations ε_{ik} on condition that $\sum S_k = 0$, one gets

$$S_k = \frac{1}{n}\sum_{i}\left(u_{ik} - \frac{1}{12}\sum_{k} u_{ik}\right) - \frac{1}{n}\sum_{i} X_{ik} \cdot a_1 - \frac{1}{n}\sum_{i}\left(X_{ik}^2 - \frac{1}{12}\sum_{k} X_{ik}^2\right) a_2$$

with

$$a_1 = \frac{n \sum_{i}\sum_{k} X_{ik} u_{ik} - \sum_{k}\left(\sum_{i} X_{ik} \sum_{i} u_{ik}\right)}{n \sum_{i}\sum_{k} X_{ik}^2 - \sum_{k}\left(\sum_{i} X_{ik}\right)^2}$$

$$a_2 = \frac{n \sum_{i}\sum_{k} X_{ik} u_{ik} - \sum_{k}\left(\sum_{i} X_{ik}^2 \sum_{i} u_{ik}\right)}{n \sum_{i}\sum_{k} X_{ik}^4 - \sum_{k}\left(\sum_{i} X_{ik}^2\right)^2}$$

§ 3. Seasonal fluctuations in the purchasing frequency

Using the above methods, we shall now examine the seasonal fluctuations in the purchasing frequency for liquor sold by bottle stores. The monthly purchasing frequency has been obtained by dividing the the amount of liquor sold per ration book by the size of the ration to which the holder is entitled. The size of the ration has been calculated by linear interpolation of the size of the ration at the end of each 3-month period as given by official statistics.

Monthly purchasing frequencies, ‰, for liquor sold by bottle stores.

Year	Month											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1924	763	769	794	839	794	879	818	834	814	828	834	1005
25	784	780	824	857	867	874	840	852	849	849	839	1003
26	782	796	888	836	850	887	862	852	858	855	860	1009
27	813	828	859	889	851	909	867	873	873	873	880	1020
28	830	844	893	886	887	918	884	897	886	890	890	1032
29	850	852	928	879	896	937	904	919	898	902	908	1046
30	868	871	908	930	892	952	912	915	913	921	906	1069
31	888	880	940	900	895	948	920	914	912	923	901	1085
32	870	835	900	871	870	949	800	819	833	826	821	1052
33	833	604	678	758	705	815	745	762	776	743	758	1027
34	697	695	842	726	767	853	765	793	784	778	793	1058
35	728	731	798	847	751	890	796	835	788	794	804	1078
36	743	774	790	860	826	851	818	829	814	843	801	1119
37	814	718	875	827	812	903	859	838	835	857	826	1146
38	789	821	840	920	804	938	863	852	857	809	787	1270
39	752	776	866	937	849	950	796	833	853	1080	399	843
40	520	546	715	690	526	730	659	715	609	627	624	1053
41	557	627	593	737	681	718	673	747	680	729	746	890
42	754	711	738	776	757	812	806	808	781	826	818	922
43	807	769	801	885	827	824	844	814	819	844	847	933
44	829	807	834	879	840	872	873	866	859	862	871	948
45	851	826	901	872	866	901	887	881	868	873	883	953
46	868	836	878	918	865	912	915	910	880	896	904	968

It is clear that the methods for calculating seasonal fluctuations described above cannot be used for, let us say, consumption series that show sudden marked changes as a result of a permanent change in the price of the commodity in question. Also, it is assumed that the seasonal fluctuations remain unchanged from year to year during the period under consideration. Accordingly, we shall work out the seasonal fluctuations in purchasing frequency for certain shorter periods of stable prices for liquor namely: 1) 1926-30/31, 2) 1934-37/38, 3) 1944-46.

Differences between observed values and moving averages for purchasing frequency.

Easter Sunday	Month											
	Year	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
4/4	1926	— 75	— 62	+ 30	— 23	— 10	+ 26	— 1	— 13	— 7	— 11	— 9	+ 139
17/4	27	— 58	— 44	— 14	+ 14	— 25	+ 31	— 12	— 7	— 9	— 10	— 5	+ 133
8/4	28	— 58	— 45	+ 2	— 6	— 6	+ 24	— 12	0	— 13	— 10	— 10	+ 131
31/3	29	— 53	— 52	+ 22	— 28	— 12	+ 28	— 7	+ 7	— 14	— 12	— 7	+ 130
20/4	30	— 49	— 46	— 9	+ 11	— 28	+ 31	— 10	— 9	— 12	— 4	— 18	+ 145
5/4	31	— 36	— 45	+ 15	— 25	— 29	+ 23						
	$\bar{S}_k$	— 54	— 47	— 0.5	— 0.5	— 18	+ 28	— 9	— 5	— 11	— 10	— 8	+ 135
1/4	1934	— 86	— 91	+ 55	— 63	— 25	+ 58	— 32	— 7	— 16	— 25	— 14	+ 250
21/4	35	— 83	— 83	— 18	+ 30	— 67	+ 71	— 25	— 12	— 37	— 31	— 24	+ 248
12/4	36	— 86	— 56	— 41	+ 26	— 10	+ 14	— 24	— 14	— 29	— 3	— 43	+ 273
30/3	37	— 36	—134	+ 22	— 27	— 44	+ 45	+ 1	— 20	— 29	— 10	— 44	+ 275
17/5	38	— 84	— 53	(—35)									
	$\bar{S}_k$	— 84	— 77	— 3	— 3	— 34	+ 53	— 24	— 12	— 28	— 17	— 34	+ 262

The purchasing frequencies for the months bordering on the price increases in 1932 and 1933 are thus not included in the moving averages. Also, according to the sales regulations in force on Oct. 1, 1938, »post-dated» purchases were permitted. This meant that the monthly ration could be purchased after the end of the month, though not more than six months later. This privelege has probably had the effect of accentuating seasonal fluctuations in purchasing frequency. This privelege of being able to take out the ration after the nominal expiry

of the monthly rationing period was cancelled in conjunction with a reduction in rations in October 1941. After this date the extra »Christmas litre» was also cancelled. Earlier, in November 1939, there had been an increase in taxes, the price of 40 % distilled spirits being increased from 4.95 to 7.95 kronor per litre.

For periods 1 and 2, method I (described in § 2) has been used.

No compensation has been made for the different number of days in the different months.

The table for the differences shows that there are marked fluctuations which appear to be fairly constant from year to year within each period. One sees that the holidays have considerable effect on the quantities purchased.

The greatest dispersion appears in the differences for March and April. The explanation of this can be seen by comparing the data given in the table for Easter Sunday. (Since the bottle stores are closed during Easter, the purchases for Easter must be made before Easter Thursday.) Accordingly, a common seasonal figure is given for March and April. It can also be pointed out that a labour dispute from January 14-February 7, 1937, affecting the retail bottle stores, caused a certain amount of »stocking up» during January and, as a result of supplies to the »system companies» being held up, decreased sales during February. This explains why the differences for January and February diverge from the other differences for the same months.

If the seasonal fluctuations for the two periods are compared, one finds that the seasonal fluctuations have been accentuated during the latter period. The reduction in purchasing as a result of the fall in income and increase in prices during 1932-33 appears to have been least, both absolutely and relatively, during the months when purchasing has been heavy, that is, in June and December. The numerical value for elasticity of purchasing during these months is, in other words, lower than for the other months. Incidentally, it can be pointed out that the same thing occurred with unchanged seasonal fluctuations since these components are assumed to be additive. The elasticity of demand during the different months, on the other

hand, is the same in the case of unchanged multiplicable seasonal fluctuations, that is to say, if assumption (1) is replaced by

$$\log u_{ik} = \log T_{ik} + \log S_k + \varepsilon'_{ik}.$$

Regarding period 3, one now obtains, using method II, the following differences for the period 1944-46.

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
— 20	— 48	+ 8	+ 8	— 20	+ 16	+ 11	+ 4	— 15	— 9	— 2	+ 67

The seasonal fluctuations have thus been levelled out after the wartime restrictions of October 1941 which, as mentioned previously, consisted of cancelling the right to »post-dated» purchases and cancelling the extra »Christmas litre». Even the increased purchasing frequency implies in itself a tendency towards levelling out of seasonal fluctuations.

§ 4. Price elasticities for the purchasing frequency

A. We shall now examine more closely the variations in the purchasing frequency about the periods when price changes took place. As a price variable we shall use the price per litre of 40 % distilled spirits. In 1928 and onwards the price of this commodity was 2.65 kronor per litre. On February 1, 1932, the price was increased to 3.25 kronor per litre, on July 1, 1932, to 3.75 kronor per litre and on February 1, 1933, to 4.35 kronor per litre.

Fig. 2 shows the purchasing frequencies for the years 1931 to 1933, inclusive, corrected for seasonal fluctuations, the seasonal fluctuations for the period 1926-30/31 and 1934-37/38 having been used alternatively. The corrected purchasing frequencies for the periods of stable prices during 1932 follow a more even course if the seasonal corrections take on values lying between the ones used. The high purchasing frequencies in June 1932 and January 1933 suggest that consumers have »stocked up» in anticipation of the coming increases in prices,

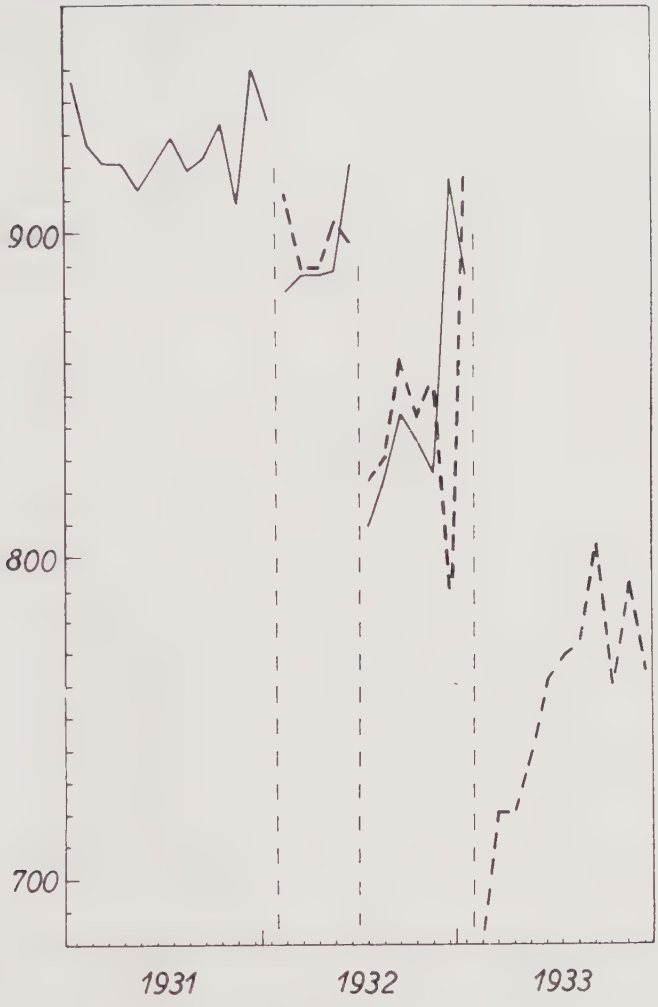

Fig. 2.

Monthly purchasing frequences (‰), corrected for seasonal fluctuations, for 1931-33.

———— : Corrections based on 1926-30/31.

— — — — : Corrections based on 1934-37/38.

which in turn has resulted in low purchasing frequencies in July 1932 and February 1933. Further, one notices that the reduction in purchasing frequency is concentrated almost wholly to the times when prices have been increased. It is conceivable,

therefore, that the effect of decreased incomes has not shown itself until plans have been made for purchasing, after an increase in prices, so that the immediate effect of such an increase is overrated.

The following values are obtained for the average monthly purchasing frequency, corrected for seasonal fluctuations, for the periods of stable prices, excluding the month immediately preceding and the month immediately following price increases. Alternative seasonal corrections have been used.

Average purchasing frequencies, ‰, corrected for seasonal fluctuations.

Period	Alt. I	Alt. II
1. 1931	925.5	—
2. March-May 1932	886.7	893.7
3. Aug.-Dec. 1932	850.2	834.4
4. March-Dec. 1933	—	760.6
5. Jan.-Sept. 1939	—	869.3
6. Jan.-Apr. 1940	—	659.5

Alt. I: Corrections based on 1926-30/31.
Alt. II: „ „ „ 1934-37/38.

The difficulties in calculating price elasticities now lie in arriving at a suitable estimate, apart from the seasonal corrections, of the incomes during the periods in question. Let us suppose that the correctly corrected purchasing frequencies for periods 2 and 3 lie between those given by alternatives I and III. The size of the income variable will be given in the following way. For 1931, the real, estimated income per inhabitant between the ages of 15 and 65 was (Chap. I, § 4) 775.0, and for 1933, 706.7 kronor. The difference between the corresponding logarithms is 0.0401. Suppose that the difference between the logarithms for average monthly incomes during periods 1 and 2 is C_1 % of the above. Suppose also that the price elasticity,

Fig. 3.

Monthly purchasing frequences in percentages for 1924-1946, corrected for seasonal fluctuations. A: Seasonal fluctuations for 1924-31. B: Seasonal fluctuations for 1933-Oct. 1941. C: Seasonal fluctuations for Oct. 1941-1946. For the period Feb. 1932-Jan. 1933 weighted averages of A and B has been used. Main increases in the price of 40 % distilled spirits: Feb. 1, 1932: 60 öre. July 1, 1932: 50 öre. Feb. 1, 1933: 60 öre. Nov. 1, 1939: 3 kronor. March 1, 1942: 1 krona. June 1, 1943: 1 krona.

Changes in rationing restrictions: Oct. 1, 1939: »Post-dated» purchases were introduced. Oct. 1, 1941: The »post-dated» purchases as well as the »Christmas litre» were cancelled; at the same time certain rations were also reduced (cf. I, § 1).

E^r_{UP}, and the income elasticity, E^r_{UI} are constant. For periods 1 and 2 one then gets

$$\log 9255 - \log 8937 - E^r_{UI} \cdot 0.000401 \cdot C_1 < E^r_{UP}(\log P_1 - \log P_2) <$$
$$< \log 9255 - \log 8867 - E^r_{UI} \cdot 0.000401 \cdot C_1$$

or, using the price for 40 % distilled spirits as a price variable

$$-0.161 + E^r_{UI} \cdot 0.0042 \cdot C_1 > E^r_{UP} > -0.197 + E^r_{UI} \cdot 0.0042 \cdot C_1.$$

In this way, and by fixing the income elasticity, one gets the following values:

$$\text{Period } 1\text{—}2\text{: } E^r_{UP} \begin{matrix} > -0.197 \\ < -0.161 \end{matrix} \Bigg\} + \begin{cases} 0.00042\, C_1; & E^r_{UI} = 0.1 \\ 0.00084\, C_1; & E^r_{UI} = 0.2 \end{cases}$$

$$\text{Period } 2\text{—}3\text{: } E^r_{UP} \begin{matrix} > -0.434 \\ < -0.266 \end{matrix} \Bigg\} + \begin{cases} 0.0012\ C_2; & E^r_{UI} = 0.2 \\ 0.0018\ C_2; & E^r_{UI} = 0.3 \end{cases}$$

$$\text{Period } 3\text{—}4\text{: } E^r_{UP} \begin{matrix} > -0.597 \\ < -0.718 \end{matrix} \Bigg\} + \begin{cases} 0.0030\ C_3; & E^r_{UI} = 0.5 \\ 0.0036\ C_3; & E^r_{UI} = 0.6 \end{cases}$$

$$\text{Period } 5\text{—}6\text{: } E^r_{UP} = \begin{matrix} -0.585; \\ -0.565; \end{matrix} \quad \begin{matrix} E^r_{UI} = 0.5 \\ E^r_{UI} = 0.6. \end{matrix}$$

Thus, assuming that the decrease in the logarithm for the average monthly income during periods 2 and 3 was, let us say, 10 % of the the decline in the logarithm for the average income during 1931 and 1933, and that the income elasticity is 0.3, one gets $-0.191 > E^r_{UP} > -0.362$. No attempt will be made here to determine the percentage values C_r. Variations in these figures cannot affect the clear tendency towards an increase in the absolute values of the price elasticities in the case of decreased purchasing frequency.

B. Let us consider the elasticity $\overline{E^r_{UP}}$ given by (cf. Chap. II, § 3, B

$$\overline{E^r_{UP}} \quad \frac{\log U_0 - \log U_1}{\log P_0 - \log P_1}.$$

Suppose, according to the discussion in § 8, one has

$$E_{KT} = 1$$

or

$$E^r_{KP} = E^r_{UP}.$$

According to II, § 3, (5) one then has

$$\frac{d \log U}{d \log P} = E^r_{KP} = M\ E\ \left(1 - \frac{F}{U}\right),$$

denoting the weighted average of the individual elasticities by $M[E]$.

If we now regard the relative number, F, of consumers, who take out their full rations as a function of the purchasing frequency U (cf. II, § 6) one gets, finally

$$E^r_{UP} = M'[E] \frac{\int_{U_0}^{U_1} d \log U}{\int_{U_0}^{U_1} d \log U \left(1 - \frac{F}{U}\right)}. \tag{1}$$

This equation should be compared with the equation (6) in II, § 3.

In order to illustrate the use of (1), suppose that the relationship between F and U is given by the function $G(U)$ in II, § 6, that is to say,

$$F = U^2 (0.43 - 1.26\,U + 1.83\,U^2).$$

Suppose, further, the seasonal fluctuations for periods 2 and 3, respectively, is given by the weighted averages of corrections I and II with weights 1 and 2, 2 and 1 respectively. One then gets the following values:

Period	U	$\frac{E^r_{UP}}{M'[E]}$	$\overline{E^r_{UP}}$
1	0.9255	— 0.27	$-0.185+\begin{cases}0.00042\ C_1; & E^r_{UI}=0.1\\ 0.00084\ C_1; & E^r_{UI}=0.2\end{cases}$
2	0.8890	— 0.38	$-0.361+\begin{cases}0.0012\ C_2; & E^r_{UI}=0.2\\ 0.0018\ C_2; & E^r_{UI}=0.3\end{cases}$
3	0.8397	— 0.49	$-0.637+\begin{cases}0.0030\ C_3; & E^r_{UI}=0.5\\ 0.0036\ C_3; & E^r_{UI}=0.6\end{cases}$
4	0.7606	—	—
5	0.8693	— 0.53	$-0.585;\quad E^r_{UI}=0.5$ $-0.565;\quad E^r_{UI}=0.6$
6	0.6595		

The values in the third column can be interpreted as follows: Suppose that the relationship between F and U is as above. Then the values in the third column are the numerical values of the elasticity $\overline{E^r_{UP}}$ one would get, if the demand is normal-elastic.

§ 5. Seasonal fluctuations and price elasticities for wine sold by bottle stores

The following table shows the differences between observed values and moving averages in the monthly quantity of wine sold (in litres per 100 ration book holders) during the periods 1925-31 and 1934-38.

It will be seen from the table that the sale of wine during December is considerably greater than during the other months. In fact, roughly 20 % of all wine sold during the year is sold during December. Also, one notices that the variations in the differences during December are large compared with the corresponding variation for the other months. There is also an indication that the seasonal fluctuation for December is on the increase. Consequently, any estimate of the seasonal fluctuations during the period in between seems to be unreliable.

For these reasons, we shall primarily concern ourselves with purchases of wine during the first eleven months of the year.

Table of differences.

Year	Month											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1925	—0.4	—5.5	—2.9	2.9	—2.0	—3.6	—6.1	—6.8	—6.4	—4.9	—3.0	+47.8
26	- 20	—5.9	—2.6	—2.6	—4.9	—2.9	—5.7	—7.5	—7.3	—5.6	—3.4	+52.2
27	—2.7	—6.8	—2.8	—2.8	—6.0	—0.4	—7.1	—8.4	—8.6	—7.5	—3.6	+59.2
28	—1.4	—6.7	—3.9	—3.9	—4.9	—2.1	—8.6	—7.6	- 8.7	—7.3	—3.7	+58.0
29	—2.4	—7.6	—3.1	—3.1	—5.6	—1.0	—7.6	—5.9	—9.9	—7.8	—2.2	+58.7
30	—2.4	—7.7	—3.5	—3.5	—7.1	+0.4	—8.0	—7.7	—9.0	—6.3	—3.7	+61.3
31	—2.7	—7.8	- 4.2	—4.2	—3.9							
1934	—1.9	—5.6	—2.5	—2.5	—4.5	—1.9	—7.0	—5.9	—6.3	—4.5	—1.2	+45.2
35	—2.7	—5.7	2.8	—2.8	—5.5	—1.0	—6.5	—4.9	—7.3	—4.8	—1.3	+45.9
36	—3.0	—4.6	—3.4	—3.4	—3.4	—4.4	—7.4	—6.6	—7.3	—3.4	—2.9	+48.9
37	+0.2	—8.6	—3.2	—3.2	—5.6	—2.2	—5.0	—7.2	—7.8	—4.7	—5.2	+53.1
38	—3.7	—4.7	—2.6	—2.6	—6.9	—0.5	—5.5	—6.6	—7.1	—5.3	—3.2	+50.0

Corrected differences between observed values and moving averages.

Year	Month										
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.
1925	+4.1	—1.0	+0.9	+0.9	+1.7	+0.5	—1.8	—2.4	—2.1	—0.6	+1.4
26	+2.4	—1.5	+1.8	+1.8	—0.5	+1.7	—0.9	—2.8	—2.6	—0.9	+1.3
27	+2.1	—2.1	+1.9	+1.9	—1.3	+4.7	—1.7	—3.0	—3.2	—2.1	+1.8
28	+4.0	—1.4	+1.5	+1.5	+0.5	+3.2	—3.4	—2.4	—3.4	—2.1	+1.5
29	+2.9	- 2.3	+2.2	+2.2	—0.4	+4.3	- 2.2	—0.6	- 4.5	-2.4	+3.1
30	+2.9	—2.3	+2.0	+2.0	—1.7	+5.9	—2.4	—2.2	—3.5	—1.3	+1.9
31	+2.9	—2.2	-1.5	+1.5	—0.6						
S_k	+3.0	1.9	+1.7	+1.7	0.5	+3.5	2.0	-2.4	—3.2	—1.6	+1.6
1934	+2.2	—1.5	+1.7	+1.7	—0.3	+2.2	—2.9	—1.8	—2.2	—0.4	+2.9
35	+1.4	—1.6	+1.0	+1.0	—1.3	+3.2	—2.4	—0.8	—3.2	—0.6	+2.9
36	+1.1	—0.4	+0.8	+0.8	+0.8	—0.1	—2.9	—2.2	—2.8	+1.1	+1.6
37	+4.6	—4.2	+1.3	+1.3	—1.3	+2.3	—0.4	—2.5	—3.1	—0.1	+1.6
38	+0.9	—0.0	+2.0	+2.0	—2.0	+4.2	—1.0	—2.0	—2.7	—0.7	+1.4
$\bar{S}_k$	+1.6	—1.1	+1.5	+1.5	—0.9	+2.6	—2.0	—2.0	—2.8	—0.3	+.21

The seasonal fluctuations are calculated with the aid of corrections according to § 2. The difference D_0 will in this case be the difference between the purchase in question for December and the corresponding moving average, that is to say the differences for December as given by the above table. In this way one gets the above corrected seasonal fluctuations for each of the months January to November.

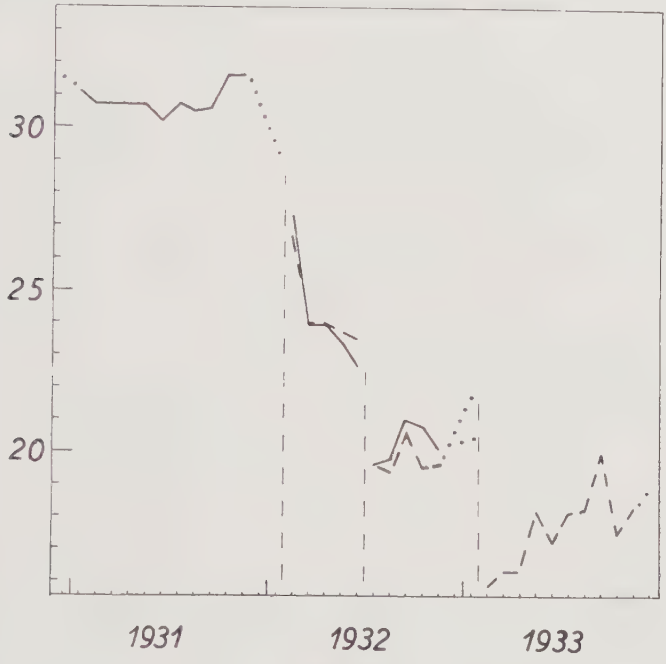

Fig. 4.

Monthly sale of wine (per 100 ration book holders), corrected for reasonal fluctuations, for 1931-33.

——— — —: Corrections based on 1925-30/31.
— — — —: Corrections based on 1934-38.

In the main, the seasonal fluctuations for the two periods seem to be practically the same.

Fig. 4 shows the monthly sale of wine per ration book holder for the period 1930-1934, corrected for seasonal fluctuations.

Calculating the monthly sale of wine per ration book holder as before, and correcting for seasonal fluctuations, but omitting the months immediately preceding and following a rise in the

price of liquor, and omitting also December, one gets, for periods of stable prices, the following values based on weighted averages of the two series of seasonal fluctuations:

Monthly sale of wine per ration book holder. With seasonal corrections.

Period 1. 1931:	30.8	Period 3. Aug.-Nov. 1932:	20.0
„ 2. Mar.-May 1932:	23.8	„ 4. Mar.-Nov. 1933:	17.8

As before, let C_ν represent the difference in question for the income logarithms, expressed as a percentage of the difference between the logarithms for income during 1931 and income during 1933.

If one sets the income elasticity at 1.27 in accordance with the results of the regression analysis, one gets, for the different increases in price, the following values for the elasticity, E_{V_1P}, of the demand for wine sold by bottle stores, with regard to the real price of 40 % distilled spirits.

$$\text{Period } 1\text{—}2: \quad E_{V_1P} = -1.19 + 0.0051\ C_1;$$
$$\text{Period } 2\text{—}3: \quad E_{V_1P} = -1.10 + 0.0076\ C_2;$$
$$\text{Period } 3\text{—}4: \quad E_{V_1P} = -0.75 + 0.0076\ C_3.$$

Regression analysis gave the price elasticity as -0.72.

As already mentioned, the sale of wine during December has been omitted from the above calculations of price elasticity. If one calculates the *total* price elasticity without correcting for seasonal fluctuations in the sale of wine, one gets the following values:

Total price elasticities of demand for wine.

Year	Nominal price	Aug.	Sept.	Oct.	Nov.	Dec.
1931-32	2.65-3.75	— 1.28	— 1.15	— 1.19	— 1.13	— 0.94
1932-33/34	3.75-4.35	— 0.46	— 0.26	— 0.71	— 0.43	— 0.08

Thus, it would seem that the price elasticity for sale of wine during December was numerically less than the elasticities for the other months.

§ 6. Seasonal fluctuations in purchases of 40% distilled spirits

As mentioned, changes of income and price affect not only the quantity of liquor purchased, but also the distribution of purchases among the different kinds of liquor. In the case of an increase in price, for example, the effect tending to reduce the quantity purchased may be partly or wholly compensated by the consumers switching over to cheaper grades of liquor.

If there were a possibility of calculating price elasticities for the expenditures, let us say E^r_{OP}, during the periods of stable prices, the difference between these elasticities would then be less than the difference between the corresponding elasticities for the purchasing frequency, E^r_{UP}. Further, one would get

$$E^r_{OP} < 1 + E^r_{UP}.$$

In this and the following paragraphs we shall give some examples of these changes. We shall, to begin with, examine the changes in purchases of 40 % distilled spirits, the cheapest and most popular type of liquor.

The series that has been used is the quantity sold by bottle stores expressed as a percentage of the total bottle store sales.

About half the liquor sold during the period 1925-38 was 40 % distilled spirits. From 1926 to 1930, the percentage fell, after wich it again increased. This increase can be attributed to the tendency to buy cheaper grades of liquor as a consequence of the purchasing power of the ration book holders beginning to weaken. During 1932 there was a marked increase in the percentage, due to price increases and a decrease in purchasing power. From 1933 to 1938, the percentage in question again fell, although the fall was gradual. This would indicate that the increased purchasing power of this period had chiefly been used in increasing the quantity of liquor purchased, while the tendency to buy inferior qualities, which had increased during 1931-33, was only gradually reversed. Accordingly, one can expect that the income elasticity for the quantity purchased will be greater for the period 1934-38 than for the

period 1925-31 (cf. Chap. I, § 4). Since 1939, purchases of 10 % distilled spirits have gone down fairly considerably, owing to deterioration in quality.

Quantities of liquor sold by bottle stores classified according to main types. Figures indicate percentages.

Year	Main type								
	40 % distilled spirits	Distilled spirits	Spirits of other kinds	96 % spirits	French brandy of cognac	Whisky	Rum and arrack	Punch	Liquer and burnt and spiced brandy
1924	68.2		7.4	0.69	19.5	1.20	0.31	2.39	0.18
25	51.6	16.7	7.5	0.59	19.8	1.11	0.24	2.32	0.18
26	50.9	16.2	7.7	0.52	21.2	1.01	0.20	2.17	0.22
27	48.6	16.8	8.3	0.45	22.4	0.99	0.18	1.98	0.25
28	47.8	16.8	10.0	0.41	21.6	0.97	0.18	1.92	0.28
29	45.8	17.0	11.6	0.37	22.0	0.97	0.18	1.80	0.29
30	46.0	17.5	11.5	0.35	21.6	0.95	0.18	1.64	0.31
31	46.9	17.3	11.0	0.32	21.8	0.85	0.17	1.46	0.30
32	54.7	15.3	9.0	0.31	18.4	0.70	0.15	1.26	0.25
33	56.6	16.3	7.9	0.26	17.0	0.59	0.14	1.06	0.21
34	55.8	17.4	7.9	0.23	16.8	0.58	0.12	1.03	0.20
35	55.9	17.5	7.8	0.20	16.8	0.57	0.11	0.99	0.20
36	55.2	18.4	7.6	0.18	16.7	0.59	0.11	0.97	0.21
37	54.0	19.3	7.8	0.17	16.8	0.63	0.11	0.95	0.22
38	54.3	19.9	7.4	0.15	16.4	0.62	0.11	0.91	0.23
Price 1930	2.66	3.08	3.22	6.02	5.55	9.80	8.49	4.86	11.68

The differences between observed values and moving averages given in the table below for the periods 1926-30/31 and 1934-37/38 have been calculated as before, the months of March and April being taken together. The differences for January and February showed a relatively large dispersion. For the years 1927, 1929, 1930 and 1937, purchases during these months were low. As far as 1937 is concerned, the low purchases can be explained by the fact that, as a result of the labour conflict referred to above, the cheaper kinds of liquor in greatest demand

were in short supply. An attempt to explain the other low figures for purchases will be made later. The low purchases occur during two consecutive months. The moving averages, and with them the differences between observed values and moving averages, have been corrected in the manner described in § 2.

Differences between observed values and moving averages for sale of 40 % distilled spirits expressed as a percentage of total bottle stores sales. Corrected differences. (Differences that have given rise to corrections are in boldface type.)

Year	Month										
	Jan.	Feb.	Mar.-Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1926	+ 0.20	— 0.23	— 0.31	+ 0.85	— 0.54	+ 0.35	+ 0.94	+ 1.69	+ 0.93	+ 1.43	— 4.79
27	**— 1.43**	**— 2.18**	— 0.96	+ 0.99	— 1.51	+ 0.16	+ 1.16	+ 1.99	+ 1.97	+ 1.21	— 5.47
28	— 0.20	+ 0.51	— 0.14	+ 0.43	— 1.08	+ 0.66	+ 0.90	+ 1.88	+ 1.86	+ 1.37	— 5.08
29	**— 0.40**	**— 0.18**	— 0.60	+ 0.46	— 1.54	+ 0.18	+ 0.83	+ 1.72	+ 1.80	+ 0.99	— 5.41
30	+ 0.36	+ 0.45	— 0.55	+ 0.75	— 1.85	+ 0.34	+ 1.08	+ 1.78	+ 1.98	+ 1.39	— 5.38
31	**— 1.29**	**— 0.54**	— 0.45	+ 1.03	— 0.77						
S_k	+ 0.21	+ 0.46	— 0.42	+ 0.70	— 1.36	+ 0.30	+ 0.98	+ 1.80	+ 1.69	+ 1.33	— 5.28
1934	— 0.96	— 0.79	— 0.61	+ 0.88	+ 0.24	+ 0.98	+ 1.48	+ 1.53	+ 0.81	— 0.61	— 3.85
35	— 0.63	— 0.40	— 0.26	+ 0.96	+ 0.48	+ 1.23	+ 1.69	+ 1.68	+ 1.29	— 0.02	— 4.93
36	— 0.63	— 0.43	— 0.63	+ 0.96	+ 0.25	+ 1.12	+ 1.97	+ 1.81	+ 1.27	— 0.13	— 5.28
37	**— 4.69**	**— 2.49**	— 0.33	+ 1.47	+ 0.26	+ 0.85	+ 1.67	+ 1.75	+ 1.18	— 0.48	— 5.90
38	— 0.54	— 0.28									
S_k	— 0.54	— 0.38	— 0.43	+ 1.00	+ 0.30	+ 1.09	+ 1.72	+ 1.76	+ 1.77	— 0.27	— 5.07

From the above table it will be seen that 40 % distilled spirits, which is cheap, accounts for a relatively smaller proportion of sales during June and December, the peak periods for sale of liquor.

When calculating the average monthly figures for the percentage in question, corrected for seasonal fluctuations, for the periods of stable prices, excluding the month immediately

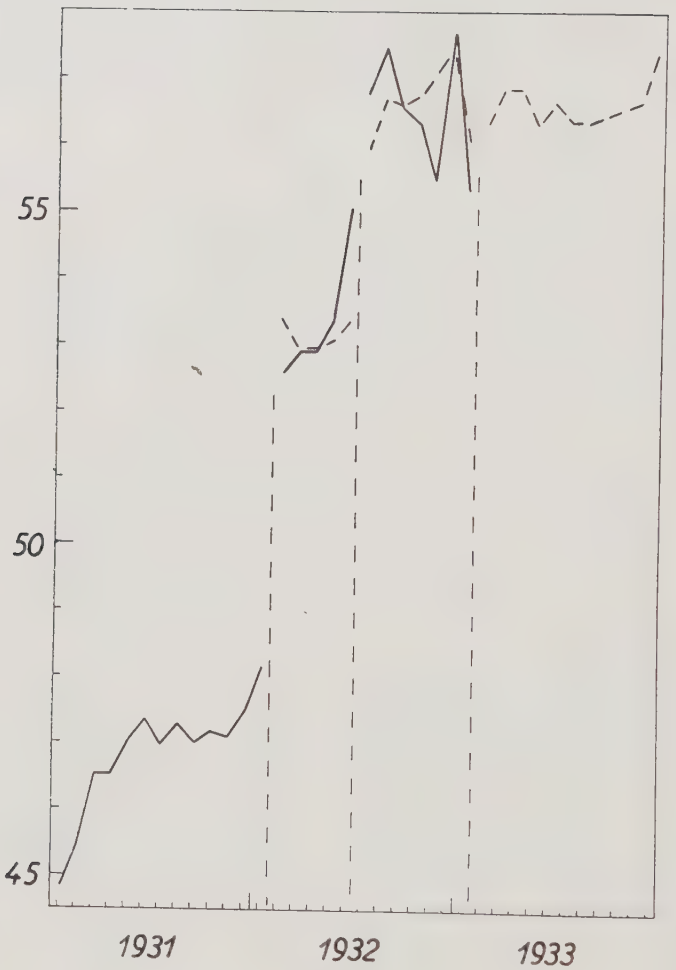

Fig. 5.

Monthly purchases of 40 % distilled spirits expressed as percentages of total bottle store sales, for 1931-33. With seasonal corrections.

———— : Corrections based on 1926-30/31.
— — — — : Corrections based on 1934-37/38.

preceding and the month immediately following price increases, one gets the following results:

Average monthly purchases of 40 % distilled spirits as percentages of total bottle stores sales. With seasonal corrections.

Period	Alt.	
	I	II
1. Mar.-Dec. 1931	47.04	—
2. Mar-May 1932	53.05	52.95
3. Aug.-Dec. 1932	56.68	56.79
4. Mar.-Dec. 1933	—	56.63

Atl. I : Corrections based on 1926-30/31.
Alt. II: „ „ „ 1934-37/38.

On examining the percentages, one finds that the increase has taken place mainly in conjunction with the first two increases in price. What is particularly noteworthy is that there has been no reaction following the increase in price in February 1933.

§ 7. Seasonal fluctuations in purchases of brandy

Unlike 40 % distilled spirits dealt with above, brandy is no uniform commodity, but consists of a number of different qualities. From 1925 to 1929 the percentage of brandy sold increased, after which it remained constant to 1931. The decrease during 1932 and 1933 would appear even in this case, for the most part at any rate, to be concentrated to occasions when prices have been increased. During the period 1934-38 the percentage of brandy sold was relatively constant.

The differences between the observed values and moving averages are given in the following table.

Here too, there was a considerable dispersion in the differences for January and February with large positive differences requiring corrections for 1927, 1929, 1931, 1935 and 1937.

These large purchases of brandy have resulted in decreased purchases of distilled spirits, although the total purchasing does not appear to have been changed. A possible explanation of the large purchases of brandy during these months is given by comparing the seasonal differences with the number of

Differences between observed values and moving averages for sale of brandy expressed as a percentage of total bottle store sales. Corrected differences. (Differences that have given rise to corrections are in boldface type.)

Year	Month										
	Jan.	Feb.	Mar.-Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1925	— 0.66	— 0.58	+ 0.67	+ 0.77	+ 1.38	+ 0.02	— 1.55	— 1.63	— 1.68	— 1.67	+ 2.48
26	— 0.89	+ 0.52	+ 1.30	+ 0.05	+ 1.26	+ 0.13	— 1.48	— 1.49	— 1.28	— 1.37	+ 2.92
27	+ **2.18**	+ **3.49**	+ 1.60	— 0.74	+ 1.34	— 0.10	— 1.24	— 1.45	— 1.25	— 0.99	+ 3.46
28	— 0.51	— 0.60	+ 0.77	+ 0.13	+ 1.24	— 0.37	— 1.15	— 1.73	— 1.40	— 1.16	+ 3.34
29	+ **0.66**	+ **0.71**	+ 0.99	— 0.20	+ 1.63	— 0.34	— 1.08	— 1.22	— 1.13	— 0.89	+ 3.25
30	— 0.76	— 0.47	+ 0.81	— 0.58	+ 1.73	— 0.24	— 1.27	— 1.03	— 1.16	— 1.20	+ 2.93
31	+ **2.21**	+ **1.52**	+ 1.17	— 0.28	+ 1.15						
S_k	+ 0.69	— 0.51	+ 1.03	— 0.16	+ 1.39	— 0.15	— 1.27	— 1.43	— 1.26	— 1.16	+ 3.13
1934	+ 0.12	+ 0.19	+ 0.84	— 0.13	+ 0.38	— 0.27	— 1.12	— 0.95	— 0.56	— 0.47	+ 1.48
35	+ **0.82**	+ **0.63**	+ 0.86	— 0.09	+ 0.15	— 0.11	— 1.19	— 1.00	— 0.84	— 0.63	+ 1.36
36	+ 0.10	+ 0.42	+ 0.97	— 0.22	+ 0.47	— 0,31	— 1.43	— 0.99	— 0.75	— 0.59	+ 1.70
37	+ **1.03**	+ **1.11**	+ 0.80	— 0.46	+ 0.43	— 0.36	— 1.24	— 0.73	— 0.54	— 0.35	+ 1.91
38	+ 0.04	— 0.06									
S_k	+ 0.09	+ 0.18	+ 0.82	— 0.19	+ 0.42	— 0,30	— 1.23	— 0.98	— 0.67	— 0.55	+ 1 58

cases of colds, influenza etc. reported by doctors during the months of January-March.[1]

The following table shows the increase in purchases based on normal values for the seasonal differences.

Increase in amount of brandy purchased, (A), during January-February (in thousands of litres) and reported cases of colds, influenza etc., (B), during January-March (in thousands[1]). Cf. fig. 6.

Year	1925	26	27	28	29	30	31	32	33	34	35	36	36	38
A	— 1	+17	+148	+2	+58	— 1	+199	—	—	+1	+24	+5	+46	— 7
B	10	22	110	12	41	8	94	11	23	8	26	25	36	10

[1] S. o. s.: »Allmän hälso- och sjukvård» (Public Health section, Sweden's official statistics).

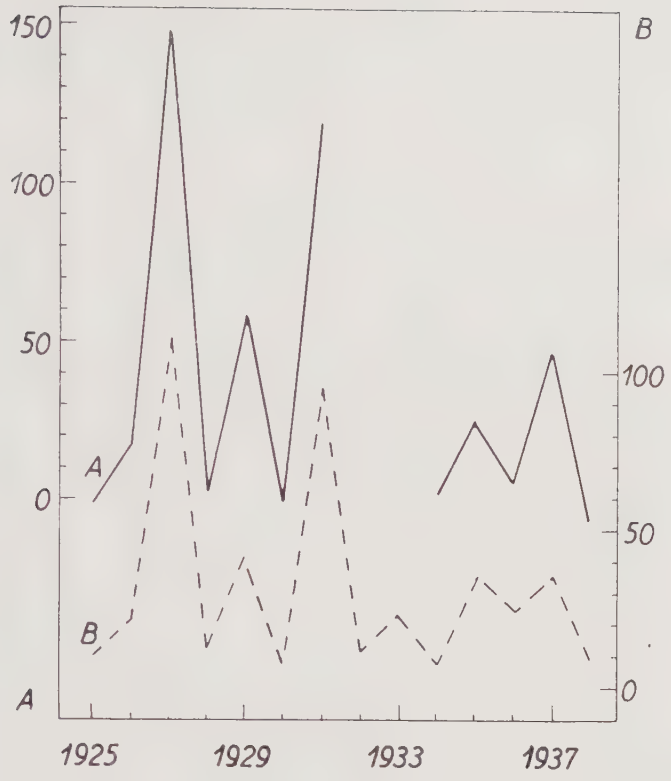

Fig. 6.

A. Increase in amount of brandy purchased during January-February 1925-1938 (in thousands of litres).

B. Reported cases of colds, influenza etc., during January-March 1925-1938 (in thousands), according to official statistics.

Monthly purchases of brandy as a percentage of total bottle store sales. With seasonal corrections.

Period	Alt.	
	I	II
1	21.20	—
2	18.87	19.02
3	17.36	17.33
4	–	16.69

The monthly percentages, (excluding the month preceding and the month following price increases) corrected for seasonal fluctuations according to the different alternatives are given in the above table.

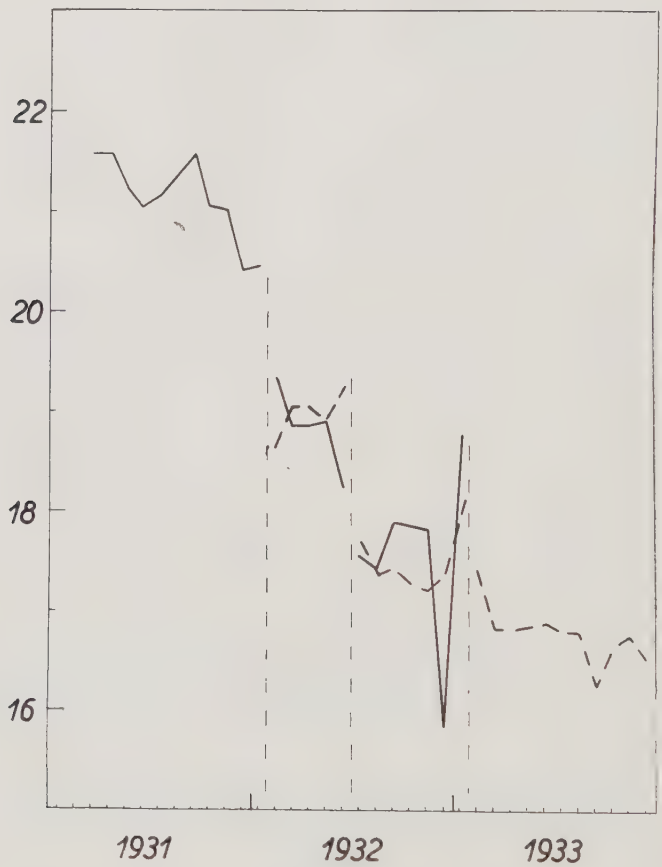

Fig. 7.

Monthly purchases of brandy, expressed as percentages of total bottle store sales, for 1931-33. With seasonal corrections.

———— : Corrections based on 1925-30/31.

— — — — : Corrections based on 1934-37/38.

CHAPTER IV

On the basis of statistics kept by the »system companies», a number of investigations have been made concerning the amount of liquor bought within different income groups.

The first and largest of these investigations deals with the period 1915-1916. During the greater part of this period, rations were much more liberal than was the case later on.

Some of the results of this investigation are given in § 1. From these results it will be seen that in the cities, purchases were greatest in the middle income groups and also that the dispersion in the quantities purchased increased with increasing incomes.

It should be pointed out that this investigation as well as the investigations mentioned below refer only to liquor sold by bottle stores, not to liquor sold on licensed premises.

The largest of the later investigations deals with 1934. During this year, prices were higher and rationing restrictions much lighter than during the period 1915-16, and consequently the amount of liquor purchased per ration book holder was lower than in 1915-16.

A description of the material for 1934 in the form of elasticities is given in § 2.

On the basis of the statistical material supplied by the investigation, a description is also given in this paragraph of the size of rations within different income groups in different city areas. The exposition is intended only to illustrate, by way of examples, certain characteristics of the factors relating to rationing and purchasing. Besides being split up according to income, the material has also been tabulated according to sex and marital status, age, and ration group. There has been no division according to social classes.[1]

[1] The Temperance Committee for 1944 will publish the results of a similar and larger investigation based on purchases made in 1944.

From this classification, it can be seen that the size of the rations shows a marked increase with the size of income. Also, the purchasing frequency is greatest for persons in the intermediate income groups. On the other hand, as mentioned previously, the intermediate income groups accounted for the greatest quantitative purchases in the 1915-16 investigation.

For a certain category of ration book holder it is possible to observe a dispersion in the quantities purchased which increases with income. This increase in dispersion with increase in income is illustrated by the fact that the relative number of ration book holders entitled to extra rations as well as the relative number of ration book holders who do not take out their full rations increases with increasing incomes. The increase in the latter case does not appear to be entirely dependent on the fact that rations increase with income.

Finally, § 3 gives certain results of a similar investigation of wine purchases.

The income elasticities are here lower than the elasticities obtained by regression analysis.

§ 1. Liquor purchased within different income groups, 1915-16

Investigations concerning liquor purchases within different income groups have been carried out on a number of different occasions.

The earliest of these investigations deals with the purchases made by certain ration book holders during the period Nov. 1, 1915 to October 31, 1916 (for certain companies, $^{1}/_{1}$ 1916-$^{31}/_{12}$ 1916).[1] The whole of the former period was unaffected by the sales regulations which came into force on Nov. 1, 1916, and which stipulated that the maximum ration for any ration book holder should be 2 litres per month. During this former period, store sales laid down by the companies. These principles,

[1] »Förbrukningen av utminuteringsvis sålt brännvin efter yrkesgrupper och inkomstklasser» (Consumption of Liquor Sold Through Bottle Stores According to Trades, Professions and Income Groups), Nykterhetskommittén X: 4. Stockholm 1919.

purchases were limited by the principles for control of bottle store sales laid down by the companies. These principles, which varied from one company to another, seem liberal when compared with later restrictions. The Stockholm liquor monopoly company, for instance, set a maximum ration of 60 litres per annum, while the company in Norrköping had a maximum ration of 72 litres per annum. In principle, customers could also obtain extra rations. In the case of the majority of the companies, purchases were checked after they had actually been made. It is worth mentioning, incidentally, that the greatest amount of liquor bought by any one person during the year covered by the investigation was 441 litres. The distribution of the quantities purchased by the holders of ration books investigated was as follows:

Distribution of purchases 1915/16.

Quantity purchased (litres)	0—10	10—25	25—50	50—100	≤ 100	Σ
Towns	2253	3624	7332	8974	345	22528
Rural areas	2021	4766	8647	4878	392	20704

Of the 43.232 ration book holders, 1.7 % bought at least 100 litres during the year. For the towns, this figure varied from 0.1 in the case of Norrköping, to 9.2 for Umeå. The figures for both companies are for 1916, when the maximum ration was set at 2 litres per month on and after November 16. That there is such a great divergency in these percentage figures can, to some extent, be accounted for by differences in control principles. When the ration book holders were split up according to sex, marital status and income, the results were as in table I.

The size of expenditure thus increases with size of income; on the other hand the quantity purchased appears, as far as the towns are concerned, to be greatest in the intermediate income groups. Further, the average price increases with income. As a rule women buy more expensive kinds of liquor than men; married men buy cheaper brands of liquor than single men.

Table I. *Average purchases in litres and kronor per ration book holder during 1915/16. Classification according to income, sex and marital status.*

Sex and marital status	Income group													
	I. Below 800 kr.		II. 800—999 kr.		III. 1000—1999 kr.		IV. 2000—2999 kr.		V. 3000—3999 kr.		VI. 4000 kr. and. above		Σ	
	Litres	Kr.	Litres	Kr.	Litres	Kr.	Litres	Kr.	Litres	Kr.	Litres	Kr.	Litres	Kr.
Towns														
Single men	36.1	67 82	42.7	77.61	43.0	82.47	38.1	89.05	32.8	91.60	37.0	117.85	40.0	83.49
Married men	41.1	72.64	45.2	77.96	45.9	81.22	46 8	90.99	40.9	94.77	42.9	126 66	44.8	89.48
Widowers	38.7	67.90	48.4	86.08	47.8	87.44	47.9	101.41	38.6	88.94	31.5	93.94	42.2	84.22
Women	21.0	44.89	22.2	49.91	17 0	41.58	14 0	35 64	15 0	38.61	16.1	44.98	18.5	43.40
Σ	35.9	65.95	43.4	76.66	44.5	80.57	43.4	89.93	37.9	91.10	39.7	118.33	42.0	85.07
Rural areas														
Single men	32.3	57.29	38,1	74.10	43.8	88.86	50.9	109.93	55.0	119.88	46.0	118.96	36.7	69.85
Married men	33.7	58.76	35.6	62.02	38.2	72.34	44.8	88.74	47.3	96.17	53.1	126.10	37.6	70.42
Widowers	32.6	59.40	33.8	66.81	38.7	75.76	39.7	81.68	46.1	96.14	57.4	145.50	38.8	69.49
Women	19.6	36.12	19.4	34.00	24.1	51.91	32 9	57 11	20 3	40.67	19.2	53 38	20.8	40 23
Σ	33.0	57.94	36.1	66.98	39.1	75.05	45.4	90.91	48.1	98.89	51.7	124.40	37.1	69.99
Towns and rural areas														
Single men	33.3	60.06	40.4	75.85	43.3	85.65	41.1	93.94	37.7	97.88	38.2	117.99	38.3	76.74
Married men	35.4	61.92	39.2	69.82	42.6	77.34	46.1	90.19	43 0	95.23	44.7	126.56	41.2	80.08
Widowers	34.9	62.58	40.9	76.23	44.1	82.67	44.4	92.92	40.7	91.00	35.1	101.15	39.2	77.29
Women	20 8	43.35	22.0	48.72	17 5	42 36	15 0	36 84	15.3	38.72	16 2	45.42	18 8	43.03
Σ	33.8	60.20	39.1	71.05	42.3	78.30	44.1	88.90	41.0	93.43	41.6	119.31	39.7	76.85

The distribution of the purchases is illustrated by the following table. It should be noted that in publishing the results of the investigation, no distinction has been made between the sexes or between married and single persons. The varying proportion of female ration book holders purchasing small quantities makes a comparison between the income groups more difficult in this respect. In the higher income groups, this proportion is low, however. On this account, only those results of the

investigation which refer to the four highest income groups in the towns and rural areas is be given here.

Distribution of purchases 1915/16. Percentages.

Towns

Quantity purchased	Income group			
	III	IV	V	VI
0— 10	6.7	8.7	14.6	17.3
10— 25	11.9	15.4	21.6	25.1
25— 50	33.5	29.5	30.7	28.1
50—100	47.5	44.5	31.2	23.1
≥100	0.4	1.9	1.9	6.3
No. of. r. b. h.	9958	2921	1060	3333
No. of. women (%)	2.7	5.2	5.1	7.1

Rural areas

Quantity purchased	Income group			
	III	IV	V	VI
0— 10	7.9	7.9	10.6	10.2
10— 25	20.7	17.9	14.5	20.3
25— 50	42.3	37.0	31.3	29.3
50—100	27.3	31.9	37.9	28.5
>100	1.8	5.3	5.7	11.7
No. of. r. b. h.	6953	1405	454	635
No. of. women (%)	0.3	0.6	0.7	2.0

There is a noticeable tendency here towards increased dispersion of quantities purchased with increased income, particularly in the case of the towns. This is shown by fact that the relative number of small and large quantities purchased appears to increase with income. However, the increase in the relative number of large quantities purchased may, to some extent, be the result of a system of control that allows persons in higher income groups the privelege of greater purchases.

§ 2. Liquor purchases within different income groups etc., 1934

In 1935, the Board of Control carried out an extensive investigation concerning liquor purchased from »system companies». The material included 27,149 ration book holders, of whom 13,594 were from towns, and the purchases investigated were made during 1934. For the ration book holders concerned, information was obtained covering liquor purchases, trade or profession, taxable income (for state and communal taxes), marital status, date of birth, number of children under 18, and size of ration. In classifying the material, communal taxation assessments were used as a basis for calculating income, except in such cases where information regarded the person's trade or profession and the relationship between both income estimates made it plain that the state taxation assessment would give a truer indication of the real income.[1] Below is given the average quantity of liquor in litres and the average value in kronor of liquor purchased per ration book holder, classified according to income, (for certain areas according to income and sex), and income and age. The material was split up into the following nine income groups.

Income group	I		No taxable income			
„	„	II	Taxable income		600— 1290	kronor
„	„	III	„	„	1300— 1990	„
„	„	IV	„	„	2000— 2990	„
„	„	V	„	„	3000— 4490	„
„	„	VI	„	„	4500— 6740	„
„	„	VII	„	„	6750— 9990	„
„	„	VIII	„	„	10000—14990	„
„	„	IX	„	„	15000	„

It was found in doing this that the average purchases, reckoned both in litres and kronor, increased with increased income.

[1] An account of the results of the investigation has been given by H. Gahn in, »Spritdrycksinköp av motboksinnehavare inom olika inkomst och åldersklasser, befolkningslager m. m.» (Purchases of Liquor by Ration Book Holders, Classified According to Income, Age, Social Categories etc.), Norrtelje, 1936.

For the majority of the urban areas, this increase was greatest in the intermediate income groups. The increase in the quantities purchased was negligable in the case of the higher income groups.

Average quantities (in litres) purchased per ration book holder in different income groups.

Area	Income group									
	I	II	III	IV	V	VI	VII	VIII	IX	Σ
Towns	18.0	18.7	21.0	26.7	33.6	34.6	35.9	37.2	39.6	28.8
Rural areas	16.5	18.5	22.1	27.0	29.4	30.7	32.0	40.6	46.4	20.4

Average expenditure (in kronor) per ration book holder in different income groups.

Area	Income group									
	I	II	III	IV	V	VI	VII	VIII	IX	Σ
Towns	90	94	103	130	166	189	217	244	315	152
Rural areas	77	87	104	130	149	168	187	265	337	97

In working out the elasticity with regard to income, figures for the lowest and highest open income group have been excluded. The average price, P, in the different income groups has been obtained by dividing the expenditure, O, by the quantity, K.

One gets then

$$E_{PI} = E_{OI} - E_{KI}.$$

It is seen from table II that the elasticity for the average price increases with increased income, while elasticity for expenditure and quantity, on the other hand, would appear to be greatest for the intermediate income groups. Also, at least in the case of the larger towns, the quantity elasticity is negligable in the higher income groups.

However, the quantity of liquor purchased is determined by both the size of ration, and the extent to which the rations

are taken out. In judging the variations in the quantities purchased, it is important, therefore, to compare them with the corresponding rations.

Table II. *Elasticities for certain intervals of income in certain urban areas.*

Income interval (kronor)	E	Area						
		Öster-malm	Söder-malm	Göte-borg	Malmö	Borås	Sunds-vall	Towns
950—2500	E_{OI}	0.14	0.22	0.59	0.18	0.21	0.08	0.32
	E_{KI}	0.12	0.23	0.62	0.24	0.24	0.12	0.36
	E_{PI}	0.02	— 0.01	— 0.03	—0.06	—0.03	—0.04	—0.03
2500—3700	E_{OI}	0.69	0.82	0.61	0.34	0.48	0.61	0.60
	E_{KI}	0.74	0.79	0.57	0.26	0.38	0.52	0.56
	E_{PI}	—0.06	0.02	0.03	0.08	0.11	0.10	0.03
2500—5600	E_{OI}	—	—	—	—	0.65	0.64	—
	E_{KI}	—	—	—	—	0.47	0.46	—
	E_{PI}	—	—	—	—	0 18	0.18	—
3700—12500	E_{OI}	0.30	0.23	0.34	0.19	0.43	0 44	0.32
	E_{KI}	0.08	0.08	0.07	— 0.03	0.26	0.25	0.09
	E_{PI}	0.22	0.15	0.27	0.22	0.17	0 19	0.23
5600—12500	E_{OI}	—	—	—	—	0.29	0.34	—
	E_{KI}	—	—	—	—	0.17	0.19	—
	E_{PI}	—	—	—	—	0.12	0.15	—

To illustrate the principles governing rationing, an investigation of the size of rations to which ration book holders are entitled has been carried out for certain large urban areas, viz: the districts of Östermalm and Södermalm in Stockholm, and the cities of Göteborg (Gothenburg) and Malmö.[1] Ration book holders for the respective areas and income groups have been split up according to sex and marital status. Further, the typical

[1] I am indebted to the Board of Control for placing the statistical material at my disposal. This material consists of a card index covering every ration book holder included in the investigation.

Table III. *Average ration (A.r.) per r.b.h. classified according to residential area, income, sex and marital status.*

Area, sex and marital status		Income group									
		I	II	III	IV	V a	V b	VI	VII	VIII	IX
Östermalm	Single men No.	—	23	14	52	34.	19	32	13	18	15
	A. r.	—	29.22	18.86	22.31	25.41	27.58	35.25	44.31	43.33	47.20
	Married men No.	38	37	58	85	104	98	160	110	108	120
	A. r.	33.30	42.48	40.55	35.53	39.12	42.00	44.15	45.27	46.57	47·50
	Single women No.	—	43	42	47	37	30	37	33		—
	A. r.	—	6.84	6.95	7.15	8.81	8.53	11.68	13.88		—
	Widows and divorced women No.	—	39	29	32	28		30	24	28	
	A. r.	—	12.21	14.07	12.75	17.57		22.93	23.33	24.57	
Södermalm	Single men No.	41	50	32	66	34	35	15	—	—	—
	A. r.	21.07	19.36	21.38	25.70	24.35	27.09	36.27	—	—	—
	Married men No.	81	81	111	218	204	264	141	34	—	—
	A. r.	25.93	32.25	58.90	23.95	40.39	41.05	42.64	43.41	—	—
	Single women No.	29	28	50	54	15		11			
	A. r.	4.42	4.64	4.72	5.33	6.67		9.82			
	Widows and divorced women No.	49	28	19	18	—	—	9	—	—	—
	A. r.	6 16	6 93	6 53	6 00	—	—	11.11	—	—	—
Göteborg	Single men No.	47	46	74	175	83	57	52	15	20	
	A. r.	17.70	17.48	20 23	20.77	23.78	25.16	26.93	40.00	43.20	
	Married men No.	98	125	245	772	780	577	431	189	109	114
	A r.	24.84	24.56	25.97	33.68	38.12	41.80	42.69	45.62	44.40	47.02
	Widowers and divorced men No.	28	10	23	39	27	10	22	14	—	—
	A. r.	18.64	17.40	20.35	29.74	35.70	36.20	45.82	46.29	—	—
	Single woman No.	—	78	101	90	33	32	34			
	A. r.	—	5.40	5.37	7,34	8.61	9.25	11.53			
	Widows and divorced women No.	—	66	55	30	21		21			
	A. r.	—	6.79	7.53	9 37	14 29		26.86			
Malmö	Married men No.	—	44	72	204	172	105	85	46	—	41
	A. r.	—	37.00	40.00	42.14	45.42	45.37	46.02	47.22	—	48.00
	Single men No.	—	- -	17	25	15	10	14	—	—	—
	A. r.		-	20.71	25.44	28.80	28.80	30.86	—	- -	—

income group V has been split up into two groups V a and V b respectively, covering the income intervals 3000-3590 kronor and 3600-4490 kronor.

The investigation gives the rations at the end of the year in such cases where the material includes ration book holders whose ration has been changed in the course of the year.

From table III it can be seen that the ration generally increases with income. On the other hand, the ration for the lower income groups is remarkably high for Östermalm. One also notices a great difference in the average ration for women, single men and married men within the same income group and residential area.

The ration book holders are further divided into the following age groups:

Age group A: Persons born 1899 or later.

Age group B: » » during period 1898-1880.

Age group C: » » 1879 or earlier.

Table IV. *Average ration for married men classified according residential area, income and age.*

Area	Age group	Income group					
		II	III	IV	V a	V b	VI
Östermalm	A	26.41	32.44	27.48	33.00	33.18	39.20
	B	44.57	39.27	37.20	40.71	43.60	44.58
	C	43.06	42.86	41.45	45.75	46.29	48.00
Södermalm	A	21.71	22.11	25.63	33.01	33.43	32.76
	B	34.12	27.57	37.63	43.31	42.72	43.80
	C	35.61	39.31	42.96	46.55	47.12	47.45
Göteborg	A	18.83	21.86	29.94	32.84	35.56	35.81
	B	28.91	25.94	34.42	40.14	43.15	43.91
	C	23.88	34 81	37.79	41.98	43.76	45.71
Malmö	A	22.00	31.20	31.24	38.12	39.60	42.86
	B	38.12	40.26	44.11	45.96	46.59	46.44
	C	38.61	44.77	45.43	46,88	47.29	47.29

Table V. *Average ration for single men, classified according to residential area, income and age.*

Östermalm	Age group	Income group					
		II	III	IV	V a	V b	VI
Östermalm	A	16.8	14.4	16.8	20.0	22.0	30 9
	B	18.8	18.0	27.0	32.4	28.0	35.1
	C	45.0	42.0	48.0	40.0	48.0	48.0
Södermalm	A	15.6	20.5	21.3	18.9	21.9	—
	B	21.4	17.0	3.33	32.0	32.0	—
	C	30.9	32.0	30.5	48.0	32.0	—
Göteborg	A	12.8	15.1	18.4	20.3	21.3	21.6
	B	17.5	119.4	25.0	25.7	27.9	31.6
	C	29.8	25.4	26.9	37.5	30.9	40.0

As may be seen from tables IV and V, age also has a great influence on the size of the rations. Rations increase the older the person is and are considerably lower in age group A than in the other age groups. Also, it can be seen that the mere fact that unmarried persons are generally younger than married men does not, in itself, account for their lower rations.

The variation in rations for different groups can also be illustrated to a certain extent by setting the variation in the number of persons with maximum rations (4 litres a month) in relation to the total number of ration book holders within that particular group. Expressed in this form, the degree of statistical reality of the variations can be examined by means of the χ^2-method.

Table VI shows the results of an analysis of this kind. The question in this case is whether the percentages within the same income group are the same for the four areas. The χ^2-analysis shows that this is not the case. The difference in the case of the lower income groups is strongly significant. In this case, Malmö has adopted a liberal rationing policy, while the number of ration book holders entitled to the maximum ration is lowest for Göteborg. In the higher income groups, the differences are smaller, owing to the small number of ration book holders in these income groups who are not entitled to maximum

Table VI. *Married, male ration book holders born 1898-1880, classified according to income, residential area and size of ration.*

Area		Income group								
		II	III	IV	V a	V b	VI	VII	VIII	Σ
Östermalm	No. of r. b. h.	11	22	40	56	60	90	66	65	410
	No. with 4 lit.	8	11	21	36	49	73	60	61	319
	„ „ (%)	72.7	50.0	52 5	64.3	81 7	81.1	90.9	93.8	
Södermalm	No. of r. b. h.	36	46	81	104	156	98	24	13	558
	No. with 4 lit.	17	11	43	74	112	75	17	12	361
	„ „ (%)	47.2	23.9	53.1	71.2	71.8	76.5	70.8	92.3	
Göteborg	No. of r. b. h.	46	109	388	445	401	295	130	69	1983
	No. with 4 lit.	9	16	107	215	281	226	119	56	1929
	„ „ (%)	19.6	14.7	27.6	48 3	70.1	76.6	91.5	81.2	
Malmö	No. of r. b. h.	17	31	111	112	68	54	32	15	440
	No. with 4 lit.	9	17	85	98	61	48	30	15	363
	„ „ (%)	52.9	54.8	76.6	87.5	89.7	88.9	93.8	100.0	
$\chi_0^{2a)}$		14 97**	27 05***	93.84***	65.62***	13.85**	4.66	10.38*	7.817*	238.0***
$\chi_0^{2b)}$		2.20	4.03*	0.004	0.80	2.23	0.59	5.74*	1.06	16.68*

a) Four areas.
b) Östermalm and Södermalm.

* $0.05 > P(\chi^2 > \chi_0^2) > 0.01$ ** $0.01 > P(\chi^2 > \chi_0^2) > 0.001$ *** $0.001 > P(\chi^2 > \chi_0^2)$

rations. Particularly interesting is a comparison between Östermalm and Södermalm. Looking at the sum total, the difference between the percentages is, to use current terminology, almost significant. Disregarding the two lowest income groups, which in the case of Östermalm show a high percentage, the difference for the other groups is not significant.

Table VII. *Purchasing frequencies* (%).
Married men.

Area	Income group							
	II	III	IV	V a	V b	VI	VII	VIII
Östermalm	91.3	89.1	94.1	93.5	97.3	90.7	82.4	78.7
Södermalm	95.5	96.1	97.2	97.1	98.4	94.5	96.0	89.6
Göteborg	89.3	89.3	91.7	92.5	91.2	88.0	85.1	81.1
Malmö	88.3	91.1	91.4	93.6	92.4	89.9	84.0	—

The results would thus indicate that rations vary considerably according to sex, marital status and age within the different income groups and also that the principles used for determining the size of the rations, if one pays due regard to the above factors, are not the same for the different rationing districts.

As a measure of the use ration book holders make of their privelege to buy liquor, it is common to take the purchasing frequency, that is to say, the relationship between the quantity purchased and the corresponding ration expressed as a percentage. This purchasing frequency has been calculated here on the basis of the nominal ration plus the extra litre in December which is accorded persons not entitled to maximum rations.

The highest purchasing frequencies are thus found among the intermediate income groups. Also, the purchasing frequency is considerably higher in Södermalm than in the other areas.

Classifying ration book holders according to sex and marital status, one finds that purchasing frequency is higher for Södermalm than for Östermalm, even in the case of single men and women.

Table VIII. *Purchasing frequencies. Classification according to sex and marital status.*

Area	Sex and material status	Income group					
		II	III	IV	V a	V b	VI
Östermalm	Single men	72.2	85.0	84.2	91.9	87.3	84.9
	Married men	91.3	89.1	94.1	93.5	97.3	90.7
	Single women	80.5	72.3	75.5	54.6	62.0	67.0
Södermalm	Single men	96.7	97.7	99.8	98.1	98.3	85.1
	Married men	95.5	96.1	97.2	97.1	98.4	94.5
	Single women	87.3	92.9	94.4	83.5		—

The extent to which the ration books have been made use of is shown also by table IX which includes married men in age group B who had a ration of 4 litres a month throughout the year covered by the investigation.

Table IX. *Number of married ration book holders in age group B classified according to income and quantities purchased.*

Area		Östermalm			Södermalm			Göteborg			Malmö		
Income group		II–IV	V	VI–VIII	II–V	V	VI–VIII	II–IV	V	VI–VIII	II–IV	V	VI–VIII
Quantities	> 48	10	18	58	18	47	17	26	82	121	7	24	7
purchased	48	11	45	51	34	108	60	40	173	70	52	72	33
(litres/year)	< 48	17	21	85	19	29	26	66	237	202	51	63	52
Percentage figures.													
Quantities	> 48	26	21	30	25	26	17	20	17	31	6	15	8
purchased	48	29	54	26	48	59	58	30	35	18	47	45	36
(litres/year)	< 48	45	25	44	27	16	25	50	48	51	46	40	57
	Σ	100	100	100	100	101	100	100	100	100	99	100	101

It will be seen from this table that the relative number of ration book holders who did not take out their full ration is, for all residential areas, lowest in the intermediate income group. Also, this figure is considerably lower for Stockholm than for Malmö and Göteborg. In the case of Göteborg, it is

highly probable that a certain number of ration book holders were prevented from making purchases owing to the nature of their work. This applies particularly to persons engaged in shipping. The relative number of ration book holders of this kind appears to be considerably greater, particularly among the intermediate income groups, in Göteborg than in the other areas included in the investigation. On this account, figures for Göteborg have sometimes been omitted from the following statistical analysis.

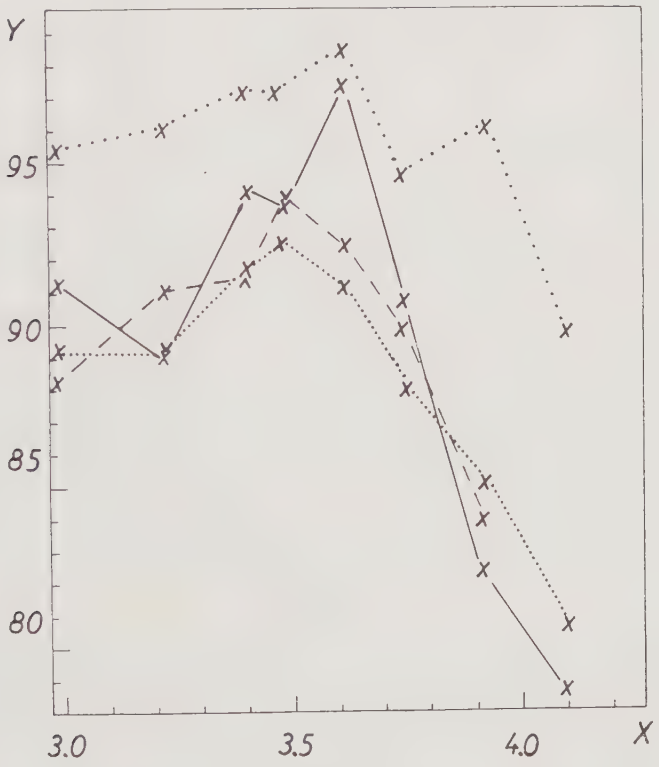

Fig. 8.

X: Log income. Y: Purchasing frequency. Married men.

———— : Östermalm. . . . . : Södermalm.

. ... : Göteborg. — — — — : Malmö.

By means of the χ^2-method we shall now go on to examine the hypothesis that the ration books in each area have been made use of to an equal extent within the different income groups. One obtains the following values:

Östermalm: $\chi^2 = 20.47^{***}$

Södermalm: $\chi^2 = 0{,}19;\ 0.1 > P > 0.05$

Göteborg: $\chi^2 = 44.32^{***}$

Malmö: $\chi^2 = 11.13^{*}$

If Göteborg is omitted, one then gets

$$\Sigma\chi^2 = 39.79^{***}$$

Certain differences in the percentage figures are thus statistically significant. It would appear that the number of ration book holders who take out their full rations is greatest in the intermediate income group. The tendency for the dispersion of the quantities purchased to increase with increasing income, a tendency noticeable in the investigation of quantities purchased during 1915/16, is also evident here. Comparing the different areas, one finds that the relative number of ration book holders in Malmö who bought more than 48 litres is extremely small. The ration book holders of this purchasing category are persons who have obtained extra rations. The more liberal rationing principles used by the Malmö company would thus, to some extent, appear to be counterbalanced by stricter rules regarding the granting of extra rations.

The question we are now faced with is this: can the difference in the use of the ration books within the intermediate and highest income groups be accounted for solely by the fact that persons in the intermediate groups have lower rations?

If we continue to limit ourselves to ration book holders entitled to 4 litres a month, then the larger ration in the case of the higher income groups can be explained by the fact that the relative number of persons with 4 litres a month is greater for the higher income groups than for the intermediate groups.

Let us now suppose that in the higher income groups we eliminate so many ration book holders who are entitled to but who do not take out maximum rations that the proportion of ration book holders entitled to 4 litres a month is the same in both income groups under examination. Now, if the relative number of ration book holders who do not take out their full rations is still greater in the higher income group, this cannot be explained by the fact that the rationing principles, in the significance attached to them here, are different for the two income groups being compared.

One gets the following results:

Percentage of married ration book holders in income group B who do not take out their full rations, classified according to income and residential area. Number reduced in the highest income group.

Area	Income group	
	V	VI—VIII
Östermalm	25.0	32.3
Södermalm	15.8	19.8
Malmö	39.6	54.5

The percentages indicate that the difference between the income groups in respect to making use of the ration books cannot be accounted for solely by the fact that the higher income group has a higher ration.

Analysing the deviations statistically, take

$$\lambda_r = \sqrt{\chi_r^2}.$$

One obtains three values for λ_r, one for each area. Finally, putting

$$\chi^2 = \left[\frac{\lambda_1 + \lambda_2 + \lambda_3}{\sqrt{3}}\right]^3$$

one gets

$$\chi^2 = 5.78^*.$$

Let us now return to the figures in table IX.

With regard to the observed differences in the percentages for the same income interval within the different areas, it is conceivable that these differences can be partly explained by the fact that the distribution of income within this interval varies for different areas. This applies particularly to the highest and lowest income groups. Consequently, it is mainly a comparison of the percentage figures for the intermediate, narrower income groups that is of interest.

An χ^2-analysis of the percentage figures for ration books holders within the same income interval who had not taken out their rations in full gives the following results:

Income groups	II—IV:	$\chi^2 = 13.00^{**}$
„ „	Va—Vb:	$\chi^2 = 66.37^{***}$
„ „	VI—VIII	$\chi^2 = 26.77^{***}$

Excluding the values for Göteborg, one gets:

Income groups	II—IV:	$\chi^2 = 7.41^{*}$
„ „	Va—Vb:	$\chi^2 = 24.27^{***}$
„ „	VI—VIII:	$\chi^2 = 20.11^{***}$

The differences examined are thus statistically significant. The relatively large number of ration books in Malmö, for instance, that had not been made full use of may conceivably be explained by the fact that a relatively large number of ration book holders in Malmö are entitled to maximum rations.

Percentage of married ration book holders in age group B who have not taken out their full rations. Classification according to income and residential area. Number reduced in the case of Östermalm and Malmö.

Area	Income group	
	V	V—VIII
Östermalm	23.2	35.9
Södermalm	15.8	25.2
Malmö	25.0	48.0

Further, one finds that:

Income groups Va—Vb: $\chi^2 = 8.01^{*}$
„ „ VI—VIII: $\chi^2 = 10.05^{**}$

Using the same method as before, it is possible to calculate the number of ration book holders who would not take out their full rations if (1) the proportion of r.b.h's for the same income groups were the same in Malmö as, let us say, in Södermalm and (2) the r.b.h's in Malmö who would now be assumed to have smaller rations, belonged to the category that did not take out the full ration of 4 litres a month.

Summing up, one can say that there are differences in the extent to which ration books are made use of, both within different income groups, and within different residential areas, where ration book holders belong to the same income interval. One can also say that these differences do not appear to be solely due to variations in the size of the ration.

§ 3. Quantities of wine purchased within different income groups

At the instigation of the board of trustees of the Swedish liquor monopoly companies, a statistical survey of liquor consumption was carried out in 1928 for use in connection with an exhibition of social conditions in Helsingfors. The survey was representative and gave figures for the amount of wine and spirits purchased in 1926 by male industrial workers in a number of towns and industrial centres, and also by the rural population in a number of »typical farming parishes».[1] As far as wine purchases are concerned, the survey gives figures only for expenditure on wine. The results are given in table I.

A later investigation of liquor purchases for 1934 carried out by the Board of Control includes, in the case of Södermalm and Östermalm (two sections of Stockholm city), figures

[1] A summary of the results of the survey is given in the »Tidskrift för systembolagen», 1928, Nos. 6 and 7. The figures in table I are taken from this source.

Table I. *Average expenditure on wine in kronor, 1926.*

Stockholm, Göteborg, Malmö and Norrköping

Income group		< 800	800-999	1000-1999	2000-2999	3000-3999	≥ 4000
Married men	No. of ration book holders	112	39	443	1416	1880	1060
	Expenditure, kr.	13.10	13.74	13.58	14.06	17.87	25.77
Single men	No. of ration book holders	86	22	231	575	554	214
	Expenditure, kr	13.40	10.46	15.99	16.46	22.09	23.77

Rural areas

Income group		< 800	800-999	1000-1999	2000-2999	3000-3999	> 2000
Married men	No. of ration book holders	1117	302	375	498	240	287
	Expenditure, kr.	2.83	3.74	2.92	2.89	5.55	18.74
Men	No. of ration holders book	1625	516	525	590	276	334
	Expenditure, kr.	2.93	3.19	2.79	3.39	5.08	17.17

Centres with industrial workers only

Income group		> 800	800-900	1000-1999	2000-2999	3000-3999	≥ 4000
Married men	No. of ration book holders	72	12	539	1275	552	474
	Expenditure, kr.	4.66	14.11	5.50	6.55	6.74	5.94
Men	No. of ration book holders	110	19	702	1527	614	540
	Expenditure, kr.	3.87	11.96	5.74	5.77	6.84	6.20

for both the number of litres of wine bought, and the expenditure on wine in kronor for that year, by those ration book holders whose purchases were examined. In an unpublished account of certain results of the investigation worked out by the Board of Control there are, among other things, figures for expenditure on wine, number of litres purchased and

average price per litre for all ration book holders within the different income groups, as well as for all male ration book holders within the different income groups.

On the basis of the statistical material supplied by the investigation, the ration books holders in the different income groups have been classified according to sex and marital status.

Table II. *Average purchases of wine, 1934.*

Södermalm

Income group		II	III	IV	V a	V b	VI	VII
Married men	No. of r. b. h.	83	111	220	205	265	141	34
	Expenditure, kr.	13.47	15.51	14.12	12.89	15.57	21.42	50.57
	Quantity, lit.	3.77	4.32	3.95	3.63	4.08	5.33	10.50
	Average price, kr.	3.57	3.59	3.57	3.55	3.82	4.02	4.82
Single men	No. of r. b. h.	53	32	66	35	35	15	—
	Expenditure, kr.	14.81	19.43	15.47	17.64	22.24	12.34	—
	Quantity, lit.	4.45	5.41	4.50	5.03	5.29	3.07	—
	Average price, kr.	3.33	3.59	3.44	3.51	3.83	4.02	—
Single women	No. of r. b. h.	28	50	54	15		11	
	Expenditure, kr.	8.57	12.21	10.81	23.71		39.92	
	Quantity, lit.	2.29	3.17	2.73	5.93		8.45	
	Average price, kr.	3.75	3.85	3.96	4.00		4.72	

Östermalm

Income group		II	III	IV	V	VI	VII	VIII	IX
Married men	No. of r. b. h.	37	60	86	203	160	110	108	86
	Expenditure, kr.	72.10	33.48	35.50	34.52	54.93	88.39	133.77	256.03
	Quantity, lit.	16.46	7.82	8.34	8.12	12.36	16.43	26.04	42.87
	Average price, kr.	4.38	4.28	4.26	4.25	4.44	5.38	5.14	5.97
Single men	No. of r. b. h.	23	14	52	53	32	13	18	14
	Expenditure, kr.	14.53	26.01	19.32	27.14	59.57	62.09	69.01	205.23
	Quantity, lit	3.87	5.07	4.76	6.08	11.09	11.38	13.06	30.93
	Average price, kr.	3.75	5.13	4.06	4.47	5.37	5.45	5.29	6.64
Single women	No. of r. b. h.	43	42	47	68	37	23	—	—
	Expenditure, kr.	13.07	15.56	21.05	17.55	25.78	27.13	—	—
	Quantity, lit.	3.23	4.28	5.39	3.88	6.03	5.34	—	—
	Average price, kr.	4.04	3.63	3.89	4.56	4.28	5.08	—	—

Figure 9 shows on a double logarithmic scale the variation in expenditure on wine with income, for married men. It would seem obvious that to simply state the constant elasticities that hold for the whole of the income interval for the different areas and periods under consideration cannot be a satisfactory way of describing the material from an elasticity point of view.

A method of describing the variation of expenditure and expenditure elasticity with income has been used by Törnqvist.[1] On the basis of results obtained from cost of living investigations in Finland, it has been found that the relationship between income I and expenditure O for a particular commodity can be stated by any of the following expressions, or a combination of them

(I) $$O = c\,\frac{I}{I+a}; \quad E_{OI} = \frac{a}{I+a}; \quad (a>0;\ c>0);$$

(II) $$O = c\,\frac{I-b}{I+a}; \quad E_{OI} = \frac{a+b}{(I+a)\,(I-b)}; \quad (b>0);$$

(III) $$O = c\,I\,\frac{I-b}{I+a}; \quad E_{OI} = 1+I\,\frac{a+b}{(I+a)\,(I-b)}.$$

Expressions of type (I) are assumed to hold for necessities, type (II) for semi-luxuries and type (III) for luxuries.

What these initial findings have in common is the assumption that the elasticity of expenditure in regard to income diminishes with the income. These expression are accordingly unsuitable for describing the material in hand, from the point of view of elasticity. Nor does it seem advisable to calculate the elasticities on the assumption that they continually increase with income. The variation of elasticity with income has been illustrated therefore by working out constant elasticities for certain intervals of income. This procedure has the disadvantage, on the other hand, that these elasticities will, to a certain extent, depend on the income interval chosen.

The results of the determinations of elasticity are shown by table III.

[1] L. Törnqvist, »Ekonomisk statistik», Ekonomisk Tidskrift 1941, No. 2, pp. 216 ff.

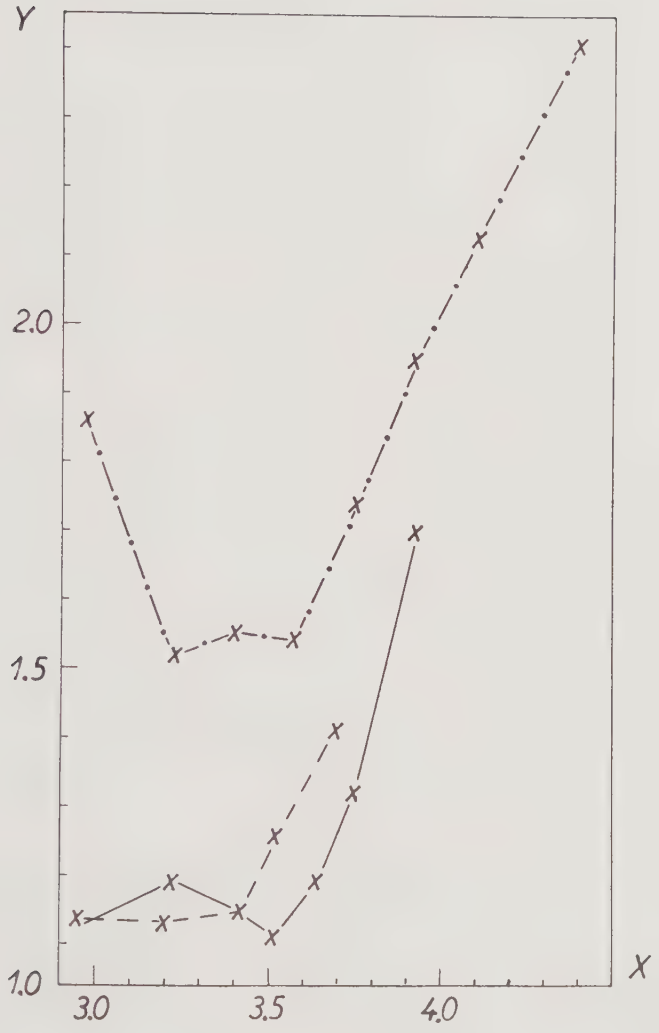

Fig. 9.

Y: Log expenditure (kr.) on wine. Married men.
X: Log income.

—.—.—.—: Östermalm 1934. ————————: Södermalm 1934.
— — — — —: Stockholm, Göteborg, Malmö and Norrköping 1926.

Table III. *Elasticities for wine purchases.*

Sex and marital status		Men			Single men			Married men		
Area	Income interval	E_{OI}	E_{KI}	E_{PI}	E_{OI}	E_{KI}	E_{PI}	E_{OI}	E_{KI}	E_{PI}
Stockholm-Göteborg-Malmö-Norrköping 1926	900-2600	0.08	—	–	0.20	–	—	0.05	—	—
Stockholm-Göteborg-Malmö-Norrköping 1926	2600-5000	0.83	—		0.59	-		0.93	—	—
Rural areas 1926	1300-4000	1.60	—				—	1.74	—	—
Södermalm 1934	950-3300	— 0.08	— 0.19	0.12	0.08	0.05	0.03	0.07	- 0.06	— 0.01
Södermalm 1934	3300-8400	1.03	0.73	0.30				1.22	0.98	0.24
Östermalm 1934	950-3700	— 0.15	— 0.15	— 0.005	0.27	0.10	0.17	0.35	0.33	— 0.02
Östermalm 1934	3700-15000	1.05	0.85	0.20	1.02	0.84	0.18	1 09	0.90	0.19

E_{OI} = The elasticity of average expenditure in regard to income.
E_{KI} The elasticity of average quantity in regard to income.
E_{PI} The elasticity of average price in regard to income.

The elasticities for the lower income interval are considerably smaller than those of the higher income interval. In the case of this latter interval, the income elasticities for quantity are slightly less than 1. Regression analysis (cf. I, § 5) gives a greater elasticity for quantity and expenditure, namely 1.27.

When judging the coefficient of elasticity, the following should be pointed out. For 1934, the ration book holders in Stockholm were split up into two categories, namely those residing in Östermalm and those residing in Södermalm. For the first group of persons, the division into social categories is considerably different to what it is in the case of the second group. Purchases of wine are both quantitatively and qualitatively higher in the case of persons living in Östermalm than for persons in the same income groups in Södermalm.

As the division which has been made does not directly correspond to a division in accordance with two social categories, it is possible that the resultant material still contains a certain amount of stratification. This would probably mean that the elasticities obtained from this material are greater than they would be if they had been calculated from material that, as far as social categories are concerned, was homogeneous.

The magnitude of the stratification effect, eliminated by splitting up, can be illustrated by comparing the expenditure elasticity for male ration book holders in Östermalm, Södermalm and Stockholm (Östermalm + Södermalm).

A r e a	Income group	E_{OI}
Södermalm	V—VII	1.30
Östermalm	V—VII	1.17
Östermalm + Södermalm	V—VII	1.68

As already mentioned, only workmen were included in investigation of wine purchases covering 1926.